B.A./B.Sc. 3rd Semester (General) Examination, 2018 (CBCS)

Subject : Mathematics (General/Generic)

Paper : BMG3CC1C/MATH-GE 3

(Real Analysis)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Symbols and notation have their usual meaning.

Group–A

1. Answer *any ten* questions from the following: 2×10=20

(a) Find supremum and infimum of the set $\left\{1+\frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$.

(b) Verify Bolzano-Weierstrass theorem for the set $\left\{1+\frac{1}{n} : n \in \mathbb{N}\right\}$.

(c) Show that the sequence $\left\{1+\frac{(-1)^n}{n}\right\}$ is bounded.

(d) Show that the sequence $\left\{\frac{3n+2}{n+1}\right\}$ is monotonically increasing.

(e) If $\sum a_n$ be a convergent series of reals, then prove that $\lim_{n\to\infty} a_n = 0$.

(f) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is convergent.

(g) Use Archimedean property to show that $\lim_{n\to\infty} \frac{1}{n} = 0$.

(h) State Cauchy's second theorem on limit.

(i) Show that the series $\sum \sin\frac{1}{n}$ is divergent by comparison test.

(j) State D'Alembert's ratio test.

(k) Let $f_n(x) = x^n,\ n \in \mathbb{N},\ x \in [0,1]$. Prove that the sequence of functions $\{f_n(x)\}$ is convergent pointwise on [0, 1].

(l) Find the number of elements of the set $\{(-1)^n : n \in \mathbb{N}\}$.

(m) Define absolute convergence of series.

(n) Define Power series of a function.

(o) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n}$.

Group–B

(Answer *any four* questions) 5×4=20

2. State Cauchy's first theorem on limit. Use it to prove that
$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots+\frac{1}{\sqrt{n^2+n}}\right]=1.$ 2+3=5

3. (a) Define Cauchy sequence.
(b) Prove that every Cauchy sequence is bounded. 2+3=5

4. (a) Prove that the geometric series $\sum ar^{n-1}, a \neq 0$ is convergent iff $|r|<1$.
(b) Examine if the set $\{x \in \mathbb{R} : \cos x = 0\}$ is finite. (1+2)+2=5

5. Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is convergent. 5

6. Let $\{x_n\}$ and $\{y_n\}$ be two sequences of reals such that $x_n < y_n, \forall\, n \in \mathbb{N}$. If $\lim_{n\to\infty} x_n = l$, $\lim_{n\to\infty} y_n = m$, then prove that $l \leq m$. 5

7. (a) Find the sum of the function $\sum_{n=1}^{\infty}(\cos x)^n$ on $(0,\pi)$.
(b) Show that, $1+x+x^2+\cdots+x^n+\cdots$ converges uniformly to $\frac{1}{1-x}$ on $-a \leq x \leq a$ for $0<a<1$. 2+3=5

Group–C

(Answer *any two* questions.) 10×2=20

8. (a) Define Cluster paint of a set $S \subset \mathbb{R}$.
(b) Prove that every sub set of a countable set is countable.
(c) Verify Bolzano-Weierstrass theorem for the set $S=\left\{\frac{n}{n+1}; n \in \mathrm{N}\right\}$. 2+4+4=10

9. (a) Prove that the limit of a sequence, if it exists, is unique.
(b) Examine the convergence of the sequence $\{x_n\}$, where $x_n = 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$.
(c) Prove that $\left\{\frac{n}{n+1}\ ; n \in \mathrm{N}\right\}$ is a Cauchy sequence. 3+4+3=10

10. (a) If a sequence $\{x_n\}$ converges to l and another sequence $\{y_n\}$ converges to m, then prove that $\{3x_n+2y_n\}$ converges to $3l+2m$.
(b) If $x_1=\sqrt{2}$, $x_{n+1}=\sqrt{2+x_n}$, $x \in \mathbb{N}$, then show that $\{x_n\}$ is convergent. Also find $\lim_{n\to\infty} x_n$. 4+(5+1)=10

11. (a) Test the convergence of the series $\left(\frac{1}{2}\right)^3+\left(\frac{1\cdot 4}{2\cdot 5}\right)^3+\left(\frac{1\cdot 4\cdot 7}{2\cdot 5\cdot 8}\right)^3+\cdots$
(b) State D'Alembert's ratio test and using it, examine the convergence of $\sum_{n=1}^{\infty}\frac{n}{2^n}$. 5+(2+3)=10

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