B.A/B.Sc 3rd Semester (General) Examination, 2020 (CBCS)

Subject: Mathematics

Course: BMG3SEC11 (Logic and set)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

Answer any eight questions: 8 × 5 = 40

1. Prove that $A-(B\cup C)=(A-B)\cap(A-C)$ for any three subsets A,B,C of a universal set. [5]

2. In a class of 50 students, 15 read Physics, 20 read Chemistry and 20 read Mathematics, 3 read Physics and Chemistry, 6 read Chemistry and Mathematics and 5 read Physics and Mathematics, 7 read none of these subjects. How many students read all the subjects? [5]

3. (i) Given $A\cap B=A\cap C$, does it imply $B=C$? Justify your answer. [2]

   (ii) Define a relation ρ on $\mathbb{R}$ by $a\ \rho\ b$ if and only if $ab\geq 0$. Is ρ an equivalence relation? Justify your answer. [3]

4. A relation ρ is defined on $\mathbb{Z}$ as $a\ \rho\ b$ holds if and only if a^2+b^2 is divisible by 2, where $a,b\in\mathbb{Z}$. Prove that ρ is an equivalence relation. [5]

5. If A,B,C are three non-empty sets then prove that [5]

$$A\times(B\cap C)=(A\times B)\cap(A\times C).$$

6. Let p and q be the propositions as [1+2+2]

   p: The election is decided.

   q: The votes have been counted.

   Express each of the propositions as an English sentence

   (i) $\sim q$ (ii) $p\wedge q$ (iii) $p\rightarrow q$.

7. (i) Is $(\sim p\rightarrow p)\rightarrow p$ a tautology? Justify your answer. [3]

   (ii) If p is the statement 'it is hot today' and q is the statement 'she is coming' then give a simple verbal sentence which describes $p\rightarrow\sim q$. [2]

8. Construct the truth table for $(\sim p\wedge q)\vee p$. [5]

9. Prove that the intersection of two equivalence relations is again an equivalence relation. [5]

10. Symbolise the following: [2+3]

    (i) All men are mortal.

    (ii) Some men are clever.

B.A/B.Sc 3rd Semester (General) Examination, 2020 (CBCS)

Subject: Mathematics

Course: BMG3SEC12 (Analytical Geometry)

Time: 2 Hours Full Marks: 40

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[Notation and Symbols have their usual meaning]

Answer *any eight* questions: 8×5 = 40

1. Identify the conic $y^2 + 2x - 4y + 3 = 0$ and find its vertex, focus, directrix. Also draw a rough sketch of it. [5]

2. Find the centre, eccentricity and foci of the conic $2x^2 + 3y^2 - 4x + 5y + 4 = 0$ and draw a rough sketch of it. [5]

3. (i) Find the equation of the parabola with focus (5,3) and directrix $3x + 2y + 7 = 0$. [3]

   (ii) State the reflection property of parabola. [2]

4. Find the equation of the hyperbola, referred to its axes as axes of coordinates, whose conjugate axis is 5 and the distance between the foci is 13. [5]

5. Discuss the nature of the conic represented by $4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$. [5]

6. Reduce the equation $4x^2 + 4xy + y^2 - 4x - 2y + a = 0$ to the canonical form and determine the type of the conic represented by it for different values of a. [5]

7. Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 2x - 4y + 5 = 0$, $x - 2y + 3z + 1 = 0$ is a great circle. [5]

8. A sphere of radius r passes through the origin and meets the axes in A, B, C. Show that the locus of the centroid of the triangle ABC in the sphere is $9(x^2 + y^2 + z^2) = 4r^2$. [5]

9. Find the equation of the right circular cylinder of radius 3, whose axis is the straight line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{2}$. [5]

10. Obtain the equation of the cylinder whose generators intersect the ellipse $9x^2 + 3y^2 = 1, z = 0$ and are parallel to the straight line with direction ratios 1, -1, 1. [5]

B.A/B.Sc. 3rd Semester (General) Examination, 2020 (CBCS)

Subject: Mathematics

Course: BMG3SEC13 (Integral Calculus)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

Answer any eight questions. 8×5 = 40

1. If $\int_0^x f(t)dt = x + \int_x^1 t\, f(t)dt$ then find the value of f(1). [5]

2. (i) State the fundamental theorem of calculus. [2]

   (ii) Evaluate $\int_{-2}^{2} |1-x^2|\, dx$ [3]

3. (i) Evaluate $\int \frac{x}{x^4-1} dx$. [3]

   (ii) Evaluate $\int (1+x)\log(1+x)\, dx$. [2]

4. Find the reduction formula for $\int (x^2+a^2)^n \; dx$. [5]

5. Deduce the reduction formula for $\int_0^{\pi/2} \cos^n x \; dx$. [3+2]

   Hence find $\int_0^{\pi} x\cos^4 x \; dx$.

6. Evaluate the integral $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dxdydz}{(1+x+y+z)^3}$. [5]

7. Evaluate $\iiint_V (x^2+y^2+z^2)dxdydz$ where V is the volume of the cube bounded by the co-ordinate planes and the planes $x = a$, $y = a$, $z = a$. [5]

8. Find the length of the arc of the curve $x = a\sin 2\theta(1+\cos 2\theta),\ y = a\cos 2\theta(1-\cos 2\theta)$ from the origin to any point. [5]

9. Find the area of the hypo-cycloid $(x/a)^{2/3} + (y/b)^{2/3} = 1$. [5]

10. Find the volume of the sphere and the surface area generated by the revolution of the circle $x^2 + y^2 = a^2$ about the x-axis. [2+3]