B.A/B.Sc. 3rd Semester (General) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMG3CC1C/MATH-GE3
(Real Analysis)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: $6\times5 = 30$

(a) Show that set of integers is neither bounded above nor bounded below. [5]

(b) Discuss the convergence of $\sum_{n=1} \frac{\sin nx}{n^2}$ [5]

(c) (i) Show that the sequence $\{a_n\}$, where $a_n = \sum_{k=1}^{n} \frac{1}{k!}$, is convergent. [3]

(ii) Find limit of $\sqrt[n]{n}$ as n $\to \infty$ [2]

(d) Show that every nonempty bounded below subset of reals has an infimum in the set of reals. [5]

(e) (i) Give an example of a subset of reals which contains its supremum but does not contain its infimum – justify your answer. [3]

(ii) What do you mean by monotone sequence? Give examples of them. [2]

(f) Show that the sequence $\{a_n\}$, where $a_n = \left(1+\frac{1}{n}\right)^n$, is monotonically decreasing and find its limit. [2+3]

(g) Define supremum and infimum of a subset of reals. Let A , B be two nonempty subsets of reals such that A is contained in B. What is the relation between sup A and sup B? Justify your answer. [1+4]

(h) (i) Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{x^n}{n!}$. [3]

(ii) State the order completeness property of the set of reals. [2]

2. Answer any three questions: $10\times3 = 30$

(a) (i) Define convergent sequence. Show that the sum of two convergent sequences is convergent. [1+5]

(ii) State and prove Cauchy's criterion for uniform convergence of a sequence of functions. [4]

(b) (i) If a sequence of continuous functions $\{f_n\}$ defined on $[a,b]$ is uniformly convergent [5] to a function f on $[a,b]$ then show that f is continuous on $[a,b]$.

(ii) Show that the sequence $\left\{\frac{n}{n+1}\right\}$ is convergent and also show that it is bounded. [3+2]

(c) (i) If the sequence $\{a_n\}$ converges to l, then show that $\left\{\frac{\sum_{k=1}^{n} a_k}{n}\right\}$ converges to l. Verify the validity of Bolzano – Weierstrass theorem for the set of natural numbers. [4+3]

(ii) Find the limit of $\left\{\frac{\sum_{k=1}^{n} a_k}{n}\right\}$, where $a_k = \sqrt[k]{k}$. [3]

(d) (i) Show that the sequence of functions $\{f_n(x)\}$, where $f_n(x) = \frac{x}{1+n^2x^2}$, uniformly converges in [-1, 1]. [5]

(ii) What is countable set? Give an example of a countable set with justification. [2+3]

(e) (i) Show that the sequence $\left\{\frac{x^n}{n!}\right\}$ is convergent for any finite real *x*. [4]

(ii) State and prove M'test for uniform convergence of a series of functions. [6]