B.A./B.Sc. 2nd Semester (General) Examination, 2019 (CBCS)

Subject : Mathematics

Paper : BMG2CC1B & Math-GE-2

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[*Notations and Symbols have their usual meaning.*]

1. Answer *any ten* questions: 2×10=20

(a) Write down the order and degree of the differential equation: $\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 + 2y = 0$.

(b) Find the differential equation of the curve $y = \frac{A}{x} + B$, where A and B are constants.

(c) Find the particular integral of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$.

(d) Show that the curve for which the normal at every point passes through a fixed point is a circle.

(e) Solve: $\frac{dy}{dx}.\tan y = \sin(x+y) + \sin(x-y)$

(f) When is a differential equation of first order said to be exact? Give an example of it.

(g) Find the differential equation of all parabolas whose axes are parallel to x-axis.

(h) Find the integrating factor for solving the differential equation $(x^3 + xy^4)dx + 2y^3dy = 0$.

(i) Solve $y = p\tan p + \log\cos p$, where $p \equiv \frac{dy}{dx}$.

(j) Examine if the equation $(y+z)dx + dy + dz = 0$ is integrable.

(k) If $z = (x+a)(y+b)$, where a, b are constants, then form a partial differential equation.

(l) Find the orthogonal trajectory of the straight lines passing through a fixed point (a, b).

(m) Examine if the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y^2 = 0$ is linear. Justify.

(n) If u and v are two independent solutions of the linear differential equation

$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$, where P, Q are functions of x,

then show that the Wronskian $w(u,v)$ is given by $w(u,v) = A\,e^{-\int Pdx}$ where A is a constant.

(o) Solve $xp + zq + y = 0$, p and q have their usual meaning.

2. Answer *any four* questions: 5×4=20

(a) Solve: $xy\frac{dy}{dx} - y^2 = (x+y)^2 e^{-\frac{y}{x}}$.

(b) Solve the following differential equation by the method of variation of parameters:

$y'' + 4y = tan2x$

(c) Find the singular solution of the differential equation satisfied by the family of curves $c^2 + 2cy - x^2 + 1 = 0$, where c is a parameter.

(d) Solve: $(y^2 + z^2 - x^2)dx - 2xy\,dy - 2xz\,dz = 0$

(e) Solve: $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^4e^x$

(f) Solve $(p^2 + q^2)y = qz$, by charpit's method, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

3. Answer *any two* questions: 10×2=20

(a) (i) Solve: $\frac{dx}{dt} = x + 3y, \frac{dy}{dt} = 3x + y$

(ii) Solve $y^2p - xyq = x(z - 2y)$, where p and q have their usual meaning. 5+5=10

(b) (i) If $y = x^2$ is a solution of $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$ then find its another independent solution.

(ii) Find a complete integral of $xpq + yq^2 = 1$ by Charpit's method where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 5+5=10

(c) (i) Solve: $(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$

(ii) Form a partial differential equation by eliminating the arbitrary function f from the equation $x + y + z = f(x^2 + y^2 + z^2)$. 5+5=10

(d) (i) Solve: $\frac{dx}{1+y} = \frac{dy}{1+x} = \frac{dz}{z}$

(ii) Solve: $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2\log x$ 5+5=10
