B.Sc. 2nd Semester (General) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMG2CC1B & Math GE-2

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as applicable.

[*Notation and symbols have their usual meaning.*]

1. Answer *any ten* of the following: 2×10=20

(a) Find the equation of the curve whose slope at any point (x, y) on it is xy and which passes through $(0,1)$.

(b) Solve the following system of differential equations:

$$\left.\begin{aligned}\frac{dx}{dt} &= -wy\\ \frac{dy}{dt} &= wx\end{aligned}\right\}$$

(c) Find the order and the degree of $\frac{d^2y}{dx^2} = \sqrt[3]{1 - \left(\frac{dy}{dx}\right)^4}$.

(d) Solve : $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{axy}$

(e) Solve : $\left(xy^2 + 3e^{x^{-3}}\right)dx - x^2ydy = 0$

(f) Solve : $p^2 + px = xy + y^2, p \equiv \frac{dy}{dx}$

(g) Find P.I. of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$.

(h) Solve : $\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = 0$.

(i) Use Wronskions to show that the functions x, x^2, x^3 are linearly independent.

(j) Solve : $x\frac{dy}{dx} + y = y^2 \log x$.

(k) Find the I.F. of the differential equation $(x^3 + y^3)dx + xy^2dy = 0$.

(l) Find the partial differential equation by eliminating a and b from $Z = ax + (1-a)y + b$.

(m) Eliminate the arbitrary function f from $Z = f(x^2 - y^2)$.

(n) Show that the equation $(y^2 + z^2 - x^2)dx - 2xydy - 2xzdz = 0$ is integrable.

(o) Classify the following second order linear differential equation:

$U_{xx} + 2U_{xy} + \cos^2 x\, U_{yy} + 2U_x + U_y = 0$.

2. Answer *any four* questions: 5×4=20

(a) Show that $(\cos y + y\cos x)dx + (\sin x - x\sin y)dy = 0$ is an exact differential equation. Then solve it.

(b) Obtain the general and singular solution of $py = p^2(x - b) + a\, ; p \equiv \frac{dy}{dx}; a, b$ are constants.

(c) Find the complete solution of $(D^2 + 4D + 3)y = e^{-3x}\, ; D \equiv \frac{d}{dx}$.

(d) Solve : $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$

(e) Verify that the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ is reduced to the form $\frac{\partial^2 z}{\partial_\alpha \partial_\beta} = 0$ by the transformation $\alpha = \frac{1}{2}(x + y), \beta = \frac{1}{2}(x - y)$.

(f) Find the complete integral of $zpq = p + q$ by Charpit's method.

3. Answer *any two* of the following: 10×2=20

(a) (i) Show that the differential equation of the family of circles $x^2 + y^2 + 2gx + 2fy + c = 0$ is $(1 + y_1^2)y_3 - 3y_1y_2^2 = 0$.

(ii) Find the general solution of the first order linear partial differential equation $xp + yq = z$.

(iii) Solve : $\frac{dy}{dx}\sin x - y\cos x + y^2 = 0$. 4+3+3

(b) (i) Solve the following differential equation by the method of variation of parameters:

$\frac{d^2y}{dx^2} + 4y = 4\tan 2x$

(ii) Obtain the Clairaut's form of the equation $x^2(y - px) = p^2y$. 6+4

(c) (i) Solve : $\frac{dx}{dt} = 5x + 4y;\ \frac{dy}{dt} = -x + y$

(ii) Solve : $yz^2(x^2 - y^2)dx + zx^2(y^2 - zx)dy + xy^2(z^2 - xy)dz = 0$ 5+5

(d) (i) Solve : $p + 3q = 5z + \tan(y - 3x)$

(ii) Solve : $(mz - ny)p + (nx - lz)q = ly - mx$

(iii) Solve : $yzp + zxq = xy$ 4+3+3

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