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B.A./B.Sc. 1st Semester (General) Examination, 2019 (CBCS)

Subject: Mathematics

Paper: BMGICC-IA/MATH-GE-I

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Using $\in - \delta$ definition show that the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sin x$ is continuous on $\mathbb{R}$.

(b) Let $f: \mathbb{R} \to \mathbb{R}$, defined by $f(x) = x|x|$. Examine if f is differentiable at $x = 0$.

(c) Sketch the graph of the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = |x| + |x-2|, x \in \mathbb{R}$.

(d) Give the geometrical interpretation of Rolle's theorem.

(e) Show that $\sin x < x$, for $x \in \left(0, \frac{\pi}{2}\right)$.

(f) If $y = \tan^{-1} x$, then prove that $(1+x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$.

(g) Examine if Lagrange's Mean Value theorem is applicable for the function $f(x) = x(x-1)(x-2)$ in $\left[0, \frac{1}{2}\right]$.

(h) Find the extreme values of the function $f(x) = x^{\frac{1}{x}}$ in its domain.

(i) Evaluate $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$

(j) Find the equation of the tangent at $\left(1, \frac{1}{2}\right)$ on the curve $xy = 1$.

(k) Prove that the radius of curvature of a circle of radius 'a' is an invariant.

(l) Find the real asymptotes of the curve $x^3 + y^3 = 3axy$, where 'a' is a constant.

(m) Examine the curve $y^2(1+x) = x^2(1-x)$ for singular points.

(n) Is the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ symmetrical about both the co-ordinate axes? Justify your answer.

(o) If $u = f\left(\frac{y}{x}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

2. Answer *any four* questions: 5×4=20

(a) If $u = \frac{x^2y^2}{x+y}$, apply Euler's theorem to find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ and hence deduce that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x^2 y} + y^2\frac{\partial^2 u}{\partial y^2} = 6u$. 2+3=5

(b) If the line $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$, show that $(a\cos\alpha)^{\frac{m}{m-1}} + (b\sin\alpha)^{\frac{m}{m-1}} = p^{\frac{m}{m-1}}$.

(c) Find the radius of curvature of the curve $y = xe^{-x}$ at the point where y is a maximum.

(d) Expand $\log(1 + \tan x)$ using Maclaurin's theorem.

(e) If $y = e^{a\sin^{-1}x}$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$.

(f) Using Taylor's theorem, show that $1 + \frac{x}{2} - \frac{x^3}{8} < \sqrt{1+x} < 1 + \frac{x}{2}, x > 0$.

3. Answer *any two* questions: 10×2=20

(a) (i) If $lx + my = 1$ is a normal to the parabola $y^2 = 4ax$, then show that $al^3 + 2alm^2 = m^2$.

(ii) State Lagrange's Mean Value Theorem. Use it to prove $0 < \frac{1}{x}\log\left(\frac{e^x-1}{x}\right) < 1$.

5+(2+3)=10

(b) (i) Find the asymptotes of curve $y^3 - 6xy^2 + 11x^2y - 6x^3 + x + y = 0$.

(ii) Find the envelope of the lines $\frac{x}{a} + \frac{y}{b} = 1$, where $a + b = c$ (c is constant). 5+5=10

(c) (i) State and prove Euler's theorem on homogeneous function in case of two variables.

(ii) Trace the curve $y^2(2a - x) = x^3$. (2+3)+5=10

(d) (i) Prove that a conical tent of a given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base.

(ii) If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. 5+5=10
