B.A/B.Sc 6th Semester (General) Examination, 2020 (CBCS)
Subject: Mathematics

Course: BMG6DSE1B1
(Numerical Methods)

Time: 3 Hours — Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions from the following: $6\times5=30$

(a) Obtain a relation between forward difference operator Δ and shift operator E. 5

(b) (i) Prove that $\Delta^n e^x = (e-1)^n e^x,$ by taking $h = 1$ 5

(c) Evaluate $\int_0^1 \frac{dx}{1+x}$ by Simpson's $\frac{1}{3}rd$ rule by taking $h = 0.25$. 5

(d) Deduce Newton-Raphson iteration formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$. Give its geometrical interpretation. 3+2

(e) Using the following data construct a Lagrange's interpolation polynomial for a given function f, where $f(40)=15.22$, $f(45)=13.99$, $f(50)=12.62$ and $f(55)=11.13$. 5

(f) Write down an algorithm and draw a flowchart for finding the largest of three distinct numbers. 5

(g) Using Euler's method, find the value of y for x=0.03 from the differential equation $\frac{dy}{dx} = x + y$ with the initial condition y=1 when x=0, correct up to four decimal places (take h= 0.01). 5

(h) Using Gauss-Seidel method, solve the following system of linear algebraic equations:
$3x + y + z = 3, x + 4y + z = 2, 2x + y + 5z = 5,$. 5

2. Answer any three questions from the following: $10\times3=30$

(a) Apply LU decomposition method to solve the following system of linear algebraic equations:
$3x + 2y + 7z = 4, 2x + 3y + z = 5, 3x + 4y + z = 7$. 10

(b) Given that

x :	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y :	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.1$. 10

(c) Discuss Gauss-Jacobi iteration method for solving a system of linear algebraic equations. Also, show that the convergence of iteration depends upon the sufficient conditions that the system must be diagonally dominant. 7+3

(d) (i) For a given function f find the missing term in the following data:

$$f(0)=1, f(1)=3, f(2)=9, f(3)=?, f(4)=81.$$

(ii) Write down the approximate representation of $\frac{2}{3}$, correct up to four significant figures and then find its absolute error and relative error. 6+2+2

(e) (i) If $f(x)$ is a quadratic polynomial, then deduce the following relation:

$$\int_1^3 f(x)dx = \frac{1}{12}[f(0) + 22f(2) + f(4)].$$

(ii) Prove that $E[\Delta f(x)] = \Delta[Ef(x)]$. 6+4

B.A./B.Sc.6th Semester(General) Examination, 2020(CBCS)
Subject: Mathematics

Paper: BMG6DSE1B2
(Complex Analysis)

Time: 3 Hours Full Marks: 60

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Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions from the following: $6 \times 5 = 30$

(a) Define analytic function. If a function $f(z)$ is analytic in a region $G \subset \mathbb{C}$ such that $f'(z) = 0$, then prove that f is constant. 2+3

(b) Show that the function $f(z) = Rlz$, where $z = x + iy$ (Rlz is the real part of $z \in \mathbb{C}$) is continuous everywhere on $\mathbb{C}$ but is differentiable nowhere on $\mathbb{C}$. 5

(c) Consider the function $f : \mathbb{C} \to \mathbb{C}$ defined by

$$f(z) = \begin{cases} e^{-z^{-4}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

Show that $f(z)$ is not analytic at $z = 0$, although the Cauchy –Riemann equations are satisfied at $z = 0$. 5

(d) Define a harmonic function. If $f(z) = u(x,y) + iv(x,y)$ is an analytic function then prove that $u(x,y)$ and $v(x,y)$ are both harmonic functions. 5

(e) Define the radius of convergence of a power series in a complex plane. Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{z^n}{n!}$.

(f) Evaluate $\int_0^{1+i} (x^2 + iy)dz$ along the paths $y = x$ and $y = x^2$. 2+3

(g) Define $\sin z$ and $\cos z$ for a complex variable z. Hence show that (i) $\sin^2 z + \cos^2 z = 1$ and (ii) $\cos 2z = 2\cos^2 z - 1$. 2+3

(h) Using Cauchy integral formula evaluate following integrals:
(i) $\int_\gamma \frac{\sin z}{z} dz$, where $\gamma = \{z \in \mathbb{C} : |z| = 1\}$ and (ii) $\int_\gamma \frac{e^{2\pi z}}{z-a} dz$, where $\gamma = a + e^{i\theta}, 0 \le \theta \le 2\pi$ 2+3

2. Answer any three questions from the following: $10 \times 3 = 30$

(a) (i) Show that an analytic function with constant modulus is constant.

(ii) Find the analytic function of which the real part is $e^{-x}\left\{\left(x^2-y^2\right)\cos y+2xy\sin y\right\}$

(b) (i) Prove that every power series represents an analytic function inside its circle of convergence.

(ii) Examine the convergence of the power series $\sum_{n=2}^{\infty}\frac{z^n}{n\left(\log n\right)^2}$. 5+5

(c) (i) Prove that a bounded entire function is constant.

(ii) Find the Laurent series expansion of the following function $f\left(z\right)=\frac{1}{z\left(z-1\right)\left(z-2\right)}$ in

$0<|z|<1$. 5+5

(d) State and prove Cauchy Integral formula. Using this formula evaluate $\int_C \frac{z}{\left(9-z^2\right)\left(z+i\right)}$,

where C is the circle $|z|=2$. 2+6+2

(e) (i) Let $f=u+iv$ be an analytic function in a region $G\subset\mathbb{C}$. If $\arg f\left(z\right)$ is constant then show that f is constant.

(ii) Evaluate $\int_0^{2\pi}\frac{d\theta}{5+3\cos\theta}$, by the method of contour integration. 3+7

B.A/B.Sc 6th Semester (General) Examination, 2020 (CBCS)
Subject: Mathematics

Course: BMG6DSE1B3
(Linear Programming)

Time: 3 Hours | Full Marks: 60

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[Notation and Symbols have their usual meaning]

1. Answer any six questions from the following: **6×5=30**

(a) Solve graphically the following linear programming problem (L.P.P.) :

Minimize $z = 3x + 5y$

subject to $2x + 3y \geq 12$, $-x + y \leq 3$, $x \leq 4$, $y \geq 3$, $x \geq 0$. 5

(b) Define a hyper plane. Show that a hyperplane is always a convex set. 1+1+3

(c) Define basic feasible solution. Obtain the basic feasible solutions of the system of following equations:

$$x_1 + 4x_2 - x_3 = 5 \text{ and } 2x_1 + 3x_2 + x_3 = 8.$$ 2+3

(d) Prove that the dual of the dual is the primal itself. 5

(e) Define slack and surplus variables for solving a L.P.P. Using slack and surplus variables, express the following L.P.P in standard form:

Maximize $w = 3x+5y-z$,

subject to $x-y+z \leq 5$, $2x+5y+z \geq 8$ and $x, y, z \geq 0$ 2+3

(f) Show that a basic feasible solution to a linear programming problem corresponds to an extreme point of the convex set of feasible solutions. 5

(g) Show that $x_1 = 5$, $x_2 = 0$, $x_3 = -1$ is a basic solution of the following system of equations:

$$x_1 + 2x_2 + x_3 = 4 \text{ and } 2x_1 + x_2 + 5x_3 = 5.$$

Find other basic solutions, if there be any. 2+3

(h) What do you mean by 'unrestricted in sign' of a variable? If any variable of the primal problem be unrestricted in sign, then prove that the corresponding constraint of the dual problem turns into equality. 1+4

2. Answer any three questions from the following: **3×10=30**

(a) When is a linear programming problem said to have a bounded solution? If at any iteration stage of the simplex algorithm we get $z_j - c_j < 0$ for at least one j and for these $j, y_{ij} \leq 0$ for all $i = 1, 2, 3, \ldots, m$, then show that the linear programming problem admits an unbounded solution of a maximization problem. 2+8

(b) State the fundamental theorem of linear programming problem. Solve the following L.P.P:

Maximize $z = x_1 + x_2 + 3x_3$

subject to $3x_1 + 2x_2 + x_3 \leq 3,$

$2x_1 + x_2 + 2x_3 \leq 2,$

and $x_1, x_2, x_3 \geq 0.$ 10

(c) Solve the following L.P.P by Big M-method:

Maximize $z = 2x_1 + 9x_2 + x_3$

subject to $x_1 + 4x_2 + 2x_3 \geq 5,$

$3x_1 + x_2 + 2x_3 \geq 4$ and $x_1, x_2, x_3 \geq 0.$ 10

(d) Convert the following primal problem to its dual problem and then solve it:

Maximize $z=3x_1+x_2$

subject to $x_1+x_2 \leq 1$

$2x_1+2x_2 \geq 2$ and $x_1, x_2 \geq 0.$ 2+8

(e) (i) A person has two types of machines and he must have at least 2 first type of machines and 5 second type of machines. The cost of each first type machine is Rs.2000 and it requires 20 m^2 space whereas the cost of each second type machine is Rs.1500 and it requires 30 m^2 space. His capital is Rs.20000 and the available space is 220 m^2. Profit from each first type machine is Rs.70 and that from each second type machine is Rs.110. Formulate an LPP for maximizing the profit earned.

(ii) Show that all the basic feasible solutions of the system of following equations $2x+6y+2z+w=3$ and $6x+4y+4z+6w=2$ are degenerate. 5+5