B.A/B.Sc 6th Semester (General) Examination, 2020 (CBCS)
Subject: Mathematics

Course: BMG6SEC41
(Boolean Algebra)

Time: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

Answer any eight questions from the following: **8×5=40**

1. Using Karnaugh map minimize the following Boolean function

   F(A,B,C,D) = $\sum$ m(0,1,3,5,7,8,9,11,13,15) 5

2. Prove that a complete $\wedge$-semi lattice is a complete lattice if and only if it has a top element. 5
3. Prove that an order (L,$\leq$) is a Lattice if sup H and inf H exist for every finite non-empty subset H of L. 5
4. State and prove "The Isomorphism Theorem for Modular Lattices". 5
5. A committee of three persons A, B, C decides proposals by a majority of votes. A has a voting weight 3, B has a voting weight 2 and C has a voting weight 1. Design a simple circuit so that light will glow when a majority of vote is cast in favour of the proposal. 5
6. Prove that a set A is totally ordered iff every non empty finite subset of A has a least element. 5
7. Let $S = \{1,2,3, \ldots 100\}$. Let $x \leq y$ means x is a divisor of y, then prove that $(S, \leq)$ is a poset. Find the maximal and minimal elements of S. 2+3
8. Let S be the set of all divisors of n, where n is a natural number, a relation $\leq$ is defined on S such that $x \leq y$ means x is a divisor of y. Prove that $(S, \leq)$ is a poset and also find $glb(x, y)$ and $lub(x, y)$. 2+3
9. Prove that in a Boolean Algebra B the following results are hold.

   $(i)\ a + a = a$ (ii) $a + 1 = 1$ 5

10. Write the principle of duality in Boolean Algebra. Using this principle prove that

    $(a + b)' = a'.b'$. 5

B.A/B.Sc 6th Semester (General) Examination, 2020 (CBCS)
Subject: Mathematics

Course: BMG6SEC42
(Transportation and Game Theory)

Time: 2 Hours Full Marks: 40

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Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

Answer any eight questions from the following: **8×5=40**

1. Prove that the number of basic variables in a balanced transportation problem with m sources/ origins and n destinations $(m, n \geq 2)$ is at most $m + n - 1$. 5
2. Determine an initial B.F.S. of the following transportation problem by north west corner method. 5

	D_1	D_2	D_3	D_4	a_i
O_1	4	6	9	5	16
O_2	2	6	4	1	12
O_3	5	7	2	9	15
b_j	12	14	9	8	43

3. Find the initial B.F.S. of the following balanced transportation problem with the help of VAM method. 5

	D_1	D_2	D_3	D_4	a_i
O_1	15	28	13	21	18
O_2	22	15	19	14	14
O_3	16	12	14	31	13
O_4	24	23	15	30	20
b_j	16	15	10	24	65

4. Prove that in an assignment problem, if a constant is added or subtracted to every element of any row (or column) of the cost matrix $\left[c_{ij}\right]$, then an assignment that minimizes the total cost on one matrix also minimizes the total cost on the other matrix. 5
5. In a two-person zero sum game, each player simultaneously shows either one or two fingers. If the number of fingers match then the player A wins a rupee from the player B. Otherwise A pays a rupee to B. Find the pay-off matrix and solve the game. 5

6. Find the optimal assignment along with total profit from the following assignment problem. 5

	D_1	D_2	D_3	D_4	D_5
O_1	2	4	3	5	4
O_2	7	4	6	8	4
O_3	2	9	8	10	4
O_4	8	6	12	7	4
O_5	2	8	5	8	8

7. Solve the following rectangular game graphically: 5

	Player B			
Player A	1	0	4	-1
	-1	1	-2	5

8. Define (a) Saddle point, (b) Pure and mixed strategies. 5

9. Using dominance property, reduce the following pay-off matrix to 2 $\times$2 game and hence find the optimal strategies and the value of the game. 5

	Player B			
Player A	1	7	3	4
	5	6	4	5
	7	2	0	3

10. Find the optimal solution of the following transportation problem if the Initial basic feasible solution is given by $x_{11} = 5, x_{14} = 2, x_{23} = 7, x_{24} = 2, x_{32} = 8, x_{34} = 10$. 5 5

	D_1	D_2	D_3	D_4	Supply
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
Demand	5	8	7	14	34

B.A/B.Sc 6th Semester (General) Examination, 2020 (CBCS)
Subject: Mathematics

Course: BMG6SEC43
(Graph Theory)

Time: 2 Hours | Full Marks: 40

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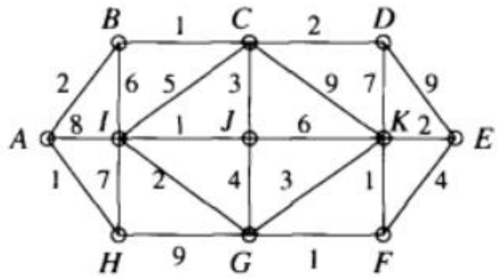
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

Answer any eight questions from the following: **8×5=40**

1. Prove that the number of odd vertices in a pseudograph is even. 5
2. If G is a simple graph with at least two vertices, prove that G must contain two or more vertices of the same degree. 5
3. A simple graph that is isomorphic to its complement is self-complementary. Prove that if a graph G is self-complementary, then G has 4k or 4k+1 vertices, where k is an integer. Further find all self-complementary graph with 5 vertices. 4+1
4. Let G be a graph and let H be a subgraph of G. Assume that H contains at least three vertices. Is it possible for G to be a complete graph and for H to be bipartite? Explain your answer. 5
5. Show that a pseudograph with at least two vertices is Eulerian if and only if it is connected and every vertex is even. 5
6. Let a graph G has $n \geq 3$ vertices and every vertex have degree at least $\frac{n}{2}$. Show that G is Hamiltonian. 5
7. Find a graph whose adjacency matrix is $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$. 5

8. Apply Dijkstra's algorithm to find the length of the shortest path from A to every other vertex in the figure. Also find the shortest path from A to E. 4+1



9. (a) Show that if G is a bipartite graph, then each cycle of g has even length.

(b) Show that if G is a graph in which the degree of each vertex is at least 2, then G contains a cycle. 2+3

10. Suppose $v_1, v_2, ..., v_n$ be a set of n vertices in a graph such that v_i and v_{i+1} are adjacent for $1 \le i \le n-1$ and, v_n and v_1 are also adjacent. Prove that for any $n \ge 4$, two isomorphic graphs must contain the same number of $n-$cycles. 5