B.A./B.Sc. 6th Semester (General) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMG6SEC41
(Boolean Algebra)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

Answer any eight questions: 8×5 = 40

1. Define a Lattice. Define a partial order in $P(X)$ so that it becomes a Lattice where X is a non-empty set. [5]
2. Prove that a poset is a chain if and only if each of its subset is a sublattice. [2+3]
3. Let $S = \{1,2,3, \dots ,100\}$. Let $x \leq y$ means x is a divisor of y. Find the maximal and minimal elements of S. 5]
4. Show that the number of all reflexive relations on a set of n elements is 2^{n^2-n}. [5]
5. In a Boolean algebra B, prove that (i) $a + a = a$, (ii) $a + (a.b) = a$, (iii) $a + I = I$ for all $a \in B$. [2+2+1]
6. Prove that there does not exist a Boolean algebra containing exactly three elements. [5]
7. Draw a circuit corresponding to the Boolean expression, $xy + yz + zx$. [5]
8. A function f is defined by $f(x, y, z) = yz + \acute{y}\acute{z}$. Find the disjunctive normal form and the conjunctive normal form of $f(x, y, z)$. [5]
9. Show that a Boolean function of two variables x_1, x_2 can be written as $f(x_1, x_2) = f(0,0)\acute{x}_1\acute{x}_2 + f(0,1)\acute{x}_1 x_2 + f(1,0)x_1\acute{x}_2 + f(1,1)x_1 x_2$. [5]
10. Define poset with an example. Show that the set of all positive divisors of 72 is a poset where the order relation is to be defined by you. [2+3]

B.A./B.Sc. 6th Semester (General) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMG6SEC42
(Transportation and Game Theory)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

Answer any eight questions: 8×5 = 40

1. State the general form of a balanced transportation problem. Show that in a transportation problem with n origin and m destination, there are (m+n-1) linearly independent equations. [1+4]

2. Show that a necessary and sufficient condition that a transportation problem has a feasible solution is [5]

$$x_{ij} = \frac{a_i b_j}{M}\left[i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, n\right]$$

where $M = \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

3. Find the initial basic feasible solution and the corresponding cost of the following problem by North-West Corner rule. [5]

	D_1	D_2	D_3	D_4	
W_1	5	6	9	4	50
W_2	3	5	7	2	50
W_3	6	4	8	2	40
	30	40	20	50	

4. Find the optimal assignment cost of the following problem by Hungarian method. [5]

	J_1	J_2	J_3	J_4
P_1	2100	4000	2500	6000
P_2	2500	3500	3500	2000
P_3	1800	3000	4000	8000
P_4	2200	5000	3000	7000

5. Apply the Least Cost rule to obtain an initial basic feasible solution of the following problem. [5]

	D_1	D_2	D_3	D_4	
W_1	5	4	2	8	10
W_2	3	5	1	6	20
W_3	4	7	3	4	10
	5	8	12	15	

6. Suppose Government requires 10, 15, 20 and 15 lakh of covid-19 vaccine doses for four districts Purulia, Birbhum, Hoogly and Howrah respectively. Also, the vaccine is available from three distribution centres - Kolkata, Howrah Station and Haldia with 25, 20 and 15 lakh doses in stock. The cost of transportation of doses from Kolkata to Purulia, Birbhum, Hoogly and Howrah are Rupees 12, 14, 10 and 9 respectively. The cost of transportation of doses from Howrah Station to Purulia, Birbhum, Hoogly and Howrah are Rupees 15, 9, 5 and 8 respectively. The cost of transportation of doses from Haldia to Purulia, Birbhum, Hoogly and Howrah are Rupees 8, 16, 9 and 11 respectively. Formulate the transportation problem. [5]

7. Solve the following transportation problem starting with the initial basic feasible solution obtained by Vogel's approximation method. [5]

	D_1	D_2	D_3	D_4	
O_1	21	16	25	13	11
O_2	17	18	14	23	13
O_3	32	17	18	41	19
	6	10	12	15	

8. Solve the two persons zero sum game graphically. [5]

	Player B			
Player A	-1	-1	2	1
	3	-2	1	-2

9. Find the initial basic feasible solution of the following problem by Vogel's approximation method. [5]

	$\boldsymbol{D_1}$	$\boldsymbol{D_2}$	$\boldsymbol{D_3}$	$\boldsymbol{D_4}$	
$\boldsymbol{W_1}$	2	9	11	6	20
$\boldsymbol{W_2}$	8	4	7	5	30
$\boldsymbol{W_3}$	7	5	6	8	20
	15	25	20	10	

10. Find the initial basic feasible solution of the following transportation problem by least cost rule. [5]

	$\boldsymbol{D_1}$	$\boldsymbol{D_2}$	$\boldsymbol{D_3}$	$\boldsymbol{D_4}$	
$\boldsymbol{W_1}$	4	7	2	3	12
$\boldsymbol{W_2}$	1	5	3	4	15
$\boldsymbol{W_3}$	2	3	6	2	10
	10	8	10	9	

B.A./B.Sc. 6th Semester (General) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMG6SEC43
(Graph Theory)

Time: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

Answer any eight questions $8\times5 = 40$

1. Prove that a connected graph is Eulerian if and only if all of its vertices are of even degree. [5]
2. If G is a simple graph with $\boldsymbol{n}$ vertices and $\boldsymbol{m}$ edges $(m\geq 3)$ such that $m \geq (1/2)(n-1)(n-2)+2$, then prove that G is Hamiltonian. Is the converse true? Support your answer. [3+2]
3. Describe the travelling-salesman problem. [5]
4. Let G be a simple graph with at most $\boldsymbol{2n}$ vertices. If degree of each vertex is at least $\boldsymbol{n}$, then show that G is connected. [5]
5. Show that, in any gathering of six people, there are three people who all know each other or three people none of whom knows either of the other two. [5]
6. Obtain a necessary and sufficient condition for a graph to be bipartite. [5]
7. If a graph has exactly two vertices of odd degree, then prove that there must be a path joining these two vertices. [5]
8. Prove that a simple graph with $\boldsymbol{n}$ vertices and $\boldsymbol{k}$ components can have at most $(n-k)(n-k+1)/2$ edges. [5]
9. Give an example in each case of a graph which is (i) Eulerian, but not Hamiltonian, (ii) Hamiltonian but not Eulerian, (iii) both Eulerian and Hamiltonian and (iv) neither Eulerian nor Hamiltonian. [5]
10. Explain Dijkstra's algorithm with an example. [5]