B.A./B.Sc. 6th Semester (General) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMG6DSE 1B1

(Numerical Methods)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) What is the rate of convergence? Write down the rate of convergence of Newton–Raphson method. 1+1

(b) The equation $x^2 + ax + b = 0$ has two real roots α and β. Show that the iteration method $x_{K+1} = -\frac{b}{x_K + a}$ is convergent near $x = \alpha$ if $|\alpha| < |\beta|$.

(c) What is the iterative formula of Regula-Falsi method? Write down the condition for convergence in the iteration $x = \phi(x)$.

(d) What are the advantage and disadvantage of bisection method.

(e) Find the quadratic polynomial which takes the following values:

x :	1	3	5
y :	24	120	336

(f) Define central difference operator.

(g) Using the definition of forward (Δ) and backward (∇) difference operators, show that

$$(1 + \Delta)(1 - \nabla) = 1$$

(h) State Newton's forward interpolation formula and its limitation.

(i) Write advantages of using forward and backward difference operators.

(j) Evaluate: $\left(\frac{\Delta^2}{E}\right) x^3$, Δ, E stand for usual notations.

(k) Give geometrical interpretation of Simpson's $\frac{1}{3}$rd rule for $\int_a^b f(x)\,dx$.

(l) If $f(0) = 1$, $f(0{\cdot}5) = 1{\cdot}5$, $f(1{\cdot}0) = 2{\cdot}0$, $f(1{\cdot}5) = 2{\cdot}5$, $f(2{\cdot}0) = 3$, then evaluate $\int_0^3 f(x)\,dx$.

(m) Given, $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$

Obtain $y(0{\cdot}3)$ taking step length 0·1.

(n) Derive trapezoidal formula of numerical integration for two points.

(o) Which method do you prefer most between Lagrange and Newton interpolation method and why?

2. Answer *any four* questions: 5×4=20

(a) Explain the Regula-Falsi method to determine approximately one simple root of an equation $f(x) = 0$.

(b) Derive the condition of convergence of the sequence of Newton-Raphson method for the solution of the equation $f(x) = 0$.

(c) Find the missing term of the following table:

x :	0	1	2	3	4
$f(x)$:	1	3	9	–	81

(d) Write down the algorithm for solving the equation $ax^2 + bx + c = 0$.

(e) Verify that, for $f(x) = 5x + 6$, the values of the integral $\int_a^{a+2h} f(x)\,dx$ obtained by Simpson's $\frac{1}{3}$ rd rule and by Trapezoidal rule with h as step length are equal. Give reasons for this equality.

(f) Prove that $\Delta^n(1/x) = \frac{(-1)^n . h^n . n!}{x(x+h)(x+2h) \ldots (x+nh)}$.

3. Answer *any two* questions: 10x2=20

(a) (i) Solve the following system of equations by LU-decomposition method:

$$x + y - z = 2$$
$$2x + 3y + 5z = -3$$
$$3x + 2y - 3z = 6$$

(ii) Describe Gauss's elimination method for numerical solution of a system of linear equations. 5+5

(b) (i) Explain the Euler's method for numerical solution of a first order differential equation $\frac{dy}{dx} = f(x, y)$ subject to the boundary condition $y = y_0$ when $x = x_0$ and comment on the accuracy of this method.

(ii) Solve by Euler's method the following differential equation for $x = 1$ by taking $h = 0{\cdot}2$

$\frac{dy}{dx} = xy$, $y = 1$ when $x = 0$. (4+2)+4

(c) (i) Explain the method of fixed point iteration of numerical solution of an equation of the form $x = \phi(x)$.

(ii) Write down the equation $x^3 + 2x - 10 = 0$ in the form $x = \phi(x)$ such that the iterative scheme about $x = 2$ converges. 5+5

(d) (i) Deduce Simpson's $\frac{1}{3}$ rd composite rule for numerical integration using Newton's forward interpolation formulae.

(ii) Show that

$$\Delta\binom{n}{x+1} = \binom{n}{x}$$

where the forward difference operator Δ operates on n and hence show that 5+5

$$\sum_{i=1}^{N}\binom{n}{i} = \binom{N+1}{i+1} - \binom{1}{i+1}.$$

B.A./B.Sc. 6th Semester (General) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMG6DSE 1B2

(Complex Analysis)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Find $\lim_{z\to 2e^{\pi i/3}}\left(\frac{z^3+8}{z^4+4z^2+16}\right)$.

(b) Find the derivative of $\frac{z-1}{z+1}$ at $z=-1-i$.

(c) Show that the function $f(z)=\begin{cases}\frac{Im(z)}{z} & ,z\neq 0\\ 0 & ,z=0\end{cases}$ is not continuous at $z=0$.

(d) Discuss the nature of discontinuity of the function $f(z)=\frac{z^2+4}{z(z-2i)}$ at the point $z=2i$.

(e) If a function $f(z)$ is continuous on a closed and bounded set $S\subset\mathbb{C}$, then show that $|f(z)|$ have maximum and minimum values of S.

(f) If a function is differentiable at a point, then show that it is continuous at that point.

(g) Show that the function $f(z)=z^3$ is entire.

(h) Prove that the sequence $z_n=-2+i\frac{(-1)^n}{n^2}$ converges to –2.

(i) State a necessary and sufficient condition for the convergence of a series of complex numbers $\sum_{n=1}^{\infty} z_n$.

(j) Find the radius and the domain of convergence of the power series $\sum_{n=0}^{\infty}\frac{1}{n^p}z^n$.

(k) Find the Laurent series of the function $f(z)=\frac{1}{z^2(1-z)}$ about $z=0$.

(l) Show that $\int_C \frac{1}{z-a}dz=2\pi i$, where C is a positively oriented closed circle centring at $z=a$.

(m) If $f(z)=u+iv$ is an entire function, then prove that u and v are both harmonic functions.

(n) Show that an analytic function with constant modulus in a connected domain is constant.

(o) Show that $\lim_{z\to 2i}\left(\frac{z^2+4}{z-2i}\right)=4i$.

2. Answer *any four* questions: 5×4=20

(a) Prove that the function $f(z) = \bar{z}$ is continuous at $z = 0$ but not differentiable at that point.

(b) Prove that $u = y^3 - 3x^2y$ is a harmonic function and find its harmonic conjugate. Also find the analytic function $f(z)$ in terms of z.

(c) Prove that the series $\sum_{n=1}^{\infty} z_n$, where $z_n = x_n + iy_n$, is convergent iff the series $\sum_{n=1}^{\infty} x_n$ and $\sum_{n=1}^{\infty} y_n$ are convergent.

(d) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{z^n}{2^n+1}$.

(e) Prove that the power series $\sum_{n=1}^{\infty} na_n z^{n-1}$ obtained by differentiating the power series $\sum_{n=1}^{\infty} a_n z^n$ has same radius of convergence.

(f) Find the Taylor series expansion of the function $f(z) = \frac{z}{z^4+9}$ around $z = 0$. Also find the radius of convergence.

3. Answer *any two* questions: 10×2=20

(a) (i) If a function $f(z) = u(x,y) + iv(x,y)$ is differentiable at a point $z_0 = x_0 + iy_0$, then show that the first order partial derivatives u_x, u_y, v_x, v_y exist at (x_0, y_0) and satisfy the equations $u_x = v_y$ and $u_y = -v_x$.

(ii) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though Cauchy-Riemann equations are satisfied thereat. 5+5

(b) (i) Find the analytic function $f(z) = u + iv$ for which the real part is $u = e^x(x\cos y - y\sin y)$.

(ii) Test the uniform convergence of the series $\sum_{n=1}^{\infty} \frac{\cos(nz)}{n^3}$ in the domain $|z| \le 1$. 5+5

(c) (i) Let $f(z)$ be analytic function throughout a disk $|z - a| < R$, where R is the radius and a is the centre. Then prove that $f(z)$ has the power series representation $f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n$, where $a_n = \frac{f^{(n)}(a)}{n!}$, $n = 0, 1, 2, \ldots$

(ii) State Cauchy integral formula and use it to evaluate the integral $\int_C \frac{\cos h(\pi z)}{z(z^2+1)} dz$, where $C: 2 + e^{i\theta}, 0 \le \theta \le 2\pi$. 5+(2+3)

(d) (i) If $f(z)$ is an analytic function of z, prove that $\left|\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right| |Re\, f(z)|^2 = 2|f'(z)|^2$.

(ii) Determine the analytic function $f(z) = u + iv$,

if $-v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cos hy)}$, and $f(\pi/2)=0$. 5+5

B.A./B.Sc. 6th Semester (General) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMG6DSE 1B3

(Linear Programming)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Is circle convex polyhedron? Justify.

(b) Define hyperplane and half-space with examples.

(c) Examine whether the set $X = \{(x_1, x_2) | x_1 \leq 4, x_2 \geq 5, x_1, x_2 \geq 0\}$ is convex or not.

(d) State the condition of unbounded solution to solve an L.P.P. by simplex Algorithm.

(e) Determine the extreme points of the set $X = \{(x, y) \in E^2 | x_1 + 2y \leq 4,\ x - y \geq 0, x \leq 5\}$.

(f) Give an example of a convex set which has infinite extreme points.

(g) Define slack and surplus variables in L.P.P.

(h) State the fundamental theorem of L.P.P.

(i) State the fundamental theorem of duality.

(j) What are the drawbacks of Big-M method?

(k) Write the dual of the following problem:
$MaxZ = 2x + 3y$
subject to $x + 2y = 5, x - 2y \leq 8, x, y \geq 0$

(l) Define a convex set with an example.

(m) Are all the boundary points of a convex set necessarily extreme points? Justify.

(n) Define basic feasible solutions of L.P.P.

(o) What is the relation between basic feasible solution and extreme point?

2. Answer *any four* questions: 5×4=20

(a) Prove that the set of all feasible solutions of an L.P.P. is a convex set.

(b) Solve graphically of the following L.P.P :
$Min\ Z = 2x_1 + 3x_2$
subject to $-x_1 + 2x_2 \leq 4, x_1 + x_2 \leq 6, x_1 + 3x_2 \geq 9$ and $x_1, x_2 \geq 0$

(c) Prove that $x_1 = 2, x_2 = 3, x_3 = 0$ is a feasible solution but not a basic feasible solution of the system of equations: $3x_1 + 5x_2 - 7x_3 = 21$, $6x_1 + 10x_2 + 3x_3 = 42$. Find the basic feasible solution or solutions of the above set of equations.

(d) Find the dual of the following L.P.P.:

$Min\ Z = x_1 + x_2 + x_3$

subject to $x_1 - 3x_2 + 4x_3 = 5$, $x_1 - 2x_2 \leq 3$, $2x_1 - x_3 \geq 0$ and $x_1, x_2 \geq 0$, x_3 is unrestricted in sign.

(e) Prove that, if a constrains of a primal problem is an equation, then the corresponding dual variable is unrestricted in sign.

(f) Find the optimal solution of the following L.P. P.:

$Min\ Z = 3x_1 + 2x_2$

subject to $7x_1 + 2x_2 \geq 30$, $5x_1 + 4x_2 \geq 20$, $2x_1 + 8x_2 \geq 16$ and $x_1, x_2 \geq 0$

3. Answer *any two* questions: 10×2=20

(a) (i) Given the L.P.P. $Max\ Z = 2x_2 + x_3$ subject to $x_1 + x_2 - 2x_3 \leq 7, -3x_1 + x_2 + 2x_3 \leq 3$ and $x_1, x_2, x_3 \geq 0$ apply simplex algorithm to prove that the problem has unbounded solution.

(ii) Prove that any point of a convex polyhedron can be expressed as convex combination of its extreme points. 7+3

(b) Solve by the Two-phase method; $Max\ Z = 5x_1 + 3x_2$ subject to $3x_1 + x_2 \leq 1$, $3x_1 + 4x_2 \geq 12$ and $x_1, x_2 \geq 0$.

(c) Find the dual of the following primal problem:

$Max\ Z = 2x_1 + 3x_2$ subject to $-x_1 + 2x_2 \leq 4$, $x_1 + x_2 \leq 6$, $x_1 + 3x_2 \leq 9$ and $x_1, x_2 \geq 0$; by solving the dual find the optimal solution of the primal problem.

(d) Solve the following L.P.P. by Big-M method:

$Maximize\ Z = 3x_1 - x_2$

subject to $-x_1 + x_2 \geq 2$,

$5x_1 - 2x_2 \geq 2$,

$x_1, x_2 \geq 0$