B.A/B.Sc 5th Semester (General) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMG5SEC31 (Probability & statistics)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

Answer any eight questions: $8\times5 = 40$

1. Define probability distribution function of a random variable X and prove that it is monotonically non-decreasing. [1+4]

2. A point P is chosen at random on a line segment AB of length *2b*. Find the expected values of (i) $AP.PB$ and (ii) $|AP - PB|$ [3+2]

3. Find the mean and variance of the distribution whose probability density function is given by $f(x) = 1 - |1 - x|, 0 \le x \le 2$ [2+3]
   $= 0$, elsewhere

4. (i) Prove that $Var(aX + b) = a^2 \text{Var}(X)$. [2]
   (ii) A continuous random variable X has the probability density function [3]
$$f(x) = \frac{1}{2} - ax, 0 \le x \le 4$$
$$= 0, \text{ elsewhere}$$
   Determine the value of a and then find $P(2 \le X \le 3)$.

5. Define normal distribution and find its moment generating function. [1+4]

6. If X is Poisson variate with parameter μ and $P(X = 0) = P(X = 1) = K$, then prove that $\mu = 1$ and $K = 1/e$. Hence calculate $P(X < 3)$. [3+2]

7. Show that the function $f(x, y)$ defined by
   $f(x, y) = \sin x \sin y, 0 < x < \frac{\pi}{2}, 0 < y < \frac{\pi}{2}$,
   $= 0$, elsewhere
   is a possible two dimensional probability density function. Find the marginal density functions. [2+3]

8. The probability density function of a continuous bivariate distribution is given by, [3+3]
$$f(x, y) = x + y \text{ for } 0 < x < 1, 0 < y < 1$$
$$= 0, \text{ elsewhere}$$
   Find the values of m_x and m_y.

9. Define binomial distribution. Find its mean and variance. [1+2+2]

10. If two independent random variables X and Y are each uniformly distributed in the interval (0,1), find the distribution of $\frac{X}{Y}$. [5]

B.A/B.Sc 5th Semester (General) Examination, 2020 (CBCS)

Subject: Mathematics

Course: BMG5SEC32 (Mathematical Finance)

Time: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

Answer any eight questions: $8\times5 = 40$

1. Define risk aversion and risk aversion factor. Write down their uses. [2+3]

2. (i) Treasury bills (or T-bills) are financial instruments that the government uses to finance public debt. Suppose you buy a 90-day T-bill with a maturity value of Rs. 10,000 for Rs. 9,800. Calculate the annual simple interest rate earned for this transaction. Round your answer to three decimal places. In all problems involving days, we assume a year has 360 days. [2]

   (ii) You are going to invest some money for one year. Bank A offers to give 6.2% interest compounded annually. Bank B offers to give 6.1% interest compounded monthly. Which is better? [3]

3. (i) An 8 % bond with 18 years to maturity has ayield of 9%. What is the price of this bond? [2]

   (ii) Find the price and duration of 10years, 8% bond that is trading at a yield of 10%? [3]

4. Suppose two competing projects (project I and project II)have cash flows of the form $(-A_1, B_1, B_1, \dots, B_1)$and $(-A_2, B_2, B_2, \dots, B_2)$ respectively both with the same length and A_1,B_1,A_2 and B_2 all are positive. Suppose $\frac{B_1}{A_1} > \frac{B_2}{A_2}$. Show that project I will have a higher IRR(internal rate of return) than project II. [5]

5. Find the convexity of a zero-coupon maturing at time t under continuous compounding (i.e., when $m \to \infty$). [2+2+1]

6. (i) What is the beta of a portfolio with an expected return of 18% if the risk-free rate is 6% and the expected market return rate is 14%? [3]

   (ii) "According to portfolio theory, systematic risk can be reduced by holding a portfolio of investments". Justify this statement whether true or false. [2]

7. Write down the differences and similarities between of NPV (net present value) and IRR (internal rate of return). [5]

8. Write a short note aboutportfolio expected return, portfolio variance and portfolio standard deviation? [2+1+2]

9. The Purple Martin has annual sales of $4,700, total debt of $1,330, total equity of $2,500 and a profit margin of 6%. What is the return on assets? [5]

10. Differentiate between systemic and non-systematic risk. Which of them can be reduced by diversification? [4+1]

B.A/B.Sc 5th Semester (General) Examination, 2020 (CBCS)

Subject: Mathematics

Course: BMG5SEC33 (Mathematical Modeling)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

Answer any eight questions: 8×5 = 40

(1) A mass of m lb is attached to the lower end of a coil spring suspended from the ceiling. The mass comes to rest in its equilibrium position, thereby stretching the spring l inch. The mass is then pulled down l_0 inch below its equilibrium position and released at t=0 with an initial velocity v_0 ft/sec., directed downward. Neglecting the resistance of the medium and assuming that no forces are present, determine the amplitude and period of the resulting motion. [5]

(2) A mass of 4 lb is attached to the lower end of a coil spring suspended from the ceiling, where the spring constant is 2 lb/ft. The mass comes to rest in its equilibrium position. It then pulled down 6 inch below its equilibrium position and released at t = 0. At this instant an external force given by $F(t) = 2\cos\omega t$ is applied to the system. Assuming the damping force in pounds is numerically equal to $3\frac{dx}{dt}$, where $\frac{dx}{dt}$ is the instantaneous velocity in ft/sec., determine the resonance frequency of the resulting motion. [5]

(3) A mass of 32 lb weight is attached to the lower end of a coil spring suspended from the ceiling. The weight comes to rest in its equilibrium position, thereby stretching the spring 2 ft. The weight is then pulled down 6 inch below its equilibrium position and released at t = 0. No external forces are present; but the resistance of the medium in pounds is numerically equal to $8\frac{dx}{dt}$, where $\frac{dx}{dt}$ is the instantaneous velocity in ft/sec. Determine the resulting motion of the weight on the spring. [5]

(4) A mass of m lb is attached to the lower end of a coil spring suspended from the ceiling, the spring constant of the spring being k lb/ft. The weight comes to rest in its equilibrium position. Beginning at t=0, an external force $F(t) = A\cos\lambda t$ is applied to the system. Determine the resulting motion if the damped force in pound is equal to two times of the instantaneous velocity in ft./sec. [5]

(5) A series-circuit has an electromotive force given by $E = 100\sin 40t\ V$, a resistor of $10\ \Omega$ and an inductor of 0.5 H. If the initial current is 0, find the current at time t (> 0). [5]

(6) The following system of differential equations describes the motions of certain pendulum: [5]

$\frac{d\theta}{dt} = y$ and

$\frac{dy}{dt} = -5\sin\theta - \frac{9}{13}y,$

where θ is the angle between the rod and the downward vertical direction and $\frac{\mathrm{d}\theta}{\mathrm{dt}} = \mathrm{y}$ is the speed at which the angle changes. Find the steady state solution for this system.

(7) A homogenous flexible string is stretched between two fixed points, (0, 0) and $(l, 0)$. The string is released from a position of rest, $u(x, 0) = \mu x(l - x)$. Through appropriate modeling, obtain an expression for the displacement $u(x, t)$ of the string at any time t. [5]

(8) Find the traffic density $\rho(x, t)$ satisfying the traffic equation [5]

$$\frac{\partial \rho}{\partial t} + (1 - 2\rho)\frac{\partial \rho}{\partial x} = 0$$

with initial traffic density as

$$\rho(x, 0) = \begin{cases} \frac{1}{4}, & x < 0 \\ \frac{1}{4}(1 - x^2)^2, & 0 \leq x < 1 \\ 0, & x \geq 1 \end{cases}$$

(9) Derive the damped wave equation of a string $u_{tt} + a\, u_t = c^2 u_{xx}$, where the damping force is proportional to the velocity, a is constant and $u(x, t)$ is the displacement of the string at (x, t). [5]

(10) The function ψ_i is defined inside a closed surface S; the function ψ_0 is defined outside S and $\nabla^2 \psi_0 = 0$. What are the conditions to be satisfied by ψ_i and ψ_0 in order that they should be the internal and external gravitational potentials of a distribution of matter inside S of density $-\frac{\nabla^2 \psi_i}{4\pi}$? [5]