B.A/B.Sc 5th Semester (General) Examination, 2021 (CBCS)

Subject: Mathematics

Course: BMG5DSE1A1

(Matrices)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: $6\times5 = 30$

(a) Prove that the intersection of any two subspaces of the real vector space $\mathbb{R}^3$ is also a subspace of $\mathbb{R}^3$. Is the union of two subspaces of $\mathbb{R}^3$ a subspace? Justify your answer. [3+2]

(b) Find the rank of the matrix : [5]

$$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}.$$

(c) Solve the following system of equations, if possible: [5]

$$x + 2y + 3z = 0$$
$$2x + 3y + 4z = 0$$
$$3x + 4y + 5z = 0.$$

(d) Diagonalize the following matrix: [5]

$$\begin{bmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{bmatrix}.$$

(e) Let $T:\mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation given by $T(x,y)=(x+3y,0,2x-4y)$. Find the matrix representation of T relative to the standard ordered bases of $\mathbb{R}^2$ and $\mathbb{R}^3$. [3+2]

(f) Find the condition that the planes given by [5]

$$bx + ay - z = 0,\ cy + bz - x = 0,\ az + cx - y = 0$$

intersect in a line.

(g) What do you mean by an idempotent matrix? Show that the eigen values of an idempotent matrix are either zero or unity. [1+4]

(h) By elementary row operations find the inverse of the following matrix: [5]

$$\begin{bmatrix} 2 & 0 & 0 \\ 4 & 3 & 0 \\ 6 & 4 & 1 \end{bmatrix}.$$

2. Answer any three questions: $10\times3 = 30$

(a) (i) What do you mean by basis of a vector space? What is the standard basis of $\mathbb{R}^3$? [1+1]

(ii) Show that the set $S = \{(x, y, z) \in \mathbb{R}^3 : 2x + y - z = 0\}$ is a subspace of $\mathbb{R}^3$. Find a basis and dimension of that subspace. [3+4+1]

(b) (i) Find the values of k for which the rank of the following matrix is 2. [5]

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & k \\ 5 & 7 & 1 & k^2 \end{bmatrix}.$$

(ii) Reduce the following matrix into its normal form and hence find its rank. [4+1]

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}.$$

(c) Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be the reflection about the x-axis, and $S:\mathbb{R}^2 \to \mathbb{R}^2$ be the linear operator that rotates each vector in the plane in the anti-clockwise direction through an angle of $\pi/2$. Find the matrix representations of T, S and TS relative to the standard ordered basis of $\mathbb{R}^2$. [3+3+4]

(d) (i) Show that a necessary and sufficient condition for a non-homogeneous system of equations $AX = B$ to be consistent is rank of A = rank of $\bar{A}$, where $\bar{A}$ is the augmented matrix of the system. [5]

(ii) Determine the values of k so that the following system of equations has (i) no solution (ii) a unique solution. [5]

$$x + y - z = 1$$
$$2x + 3y + kz = 3$$
$$x + ky + 3z = 2.$$

(e) (i) In $\mathbb{R}^3$, $\alpha = (4,3,5)$, $\beta = (0,1,3)$, $\gamma = (2,1,1)$. Examine if α is a linear combination of β and γ. Mention whether $\{\alpha, \beta, \gamma\}$ is a linearly independent subset of $\mathbb{R}^3$. [3+1]

(ii) Show that the eigen values of a real symmetric matrix are all real. [6]

B.A/B.Sc 5th Semester (General) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMG5DSE1A2
(Mechanics)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: $6\times5 = 30$

(a) Prove that, if each force of a coplanar system be rotated through an angle θ in its plane in the same sense, then their resultant is also rotated through the same angle. [5]

(b) Three forces P, Q, R act along the sides of the triangle formed by the lines $x + y = 1$, $y - x = 1$ and $y = 2$. Find the equation of the line of action of their resultant. [5]

(c) Show that the least force which will be move a weight W along a rough horizontal plane is W $\sin\lambda$, where λ is the angle of friction. [5]

(d) Find the centre of gravity of a quadrant of an ellipse. [5]

(e) A particle moves freely in a parabolic path given by $y^2 = 4ax$, under a force which is always perpendicular to its axis. Find the law of force. [5]

(f) Find the resultant of two simple harmonic motions having same periods. [5]

(g) A particle moves with a simple harmonic motion, its position of rest being at a distance '***a***' from the centre. Find, by the principle of energy, the velocity at the centre. [5]

(h) A particle moves along a straight line, the law of motion being $x = A\cos(nt + \in)$. Show that $v^2 = n^2(A^2 - x^2)$ and the acceleration is directed to the origin and varies as the distance. [2+2+1]

2. Answer any three questions: $10\times3 = 30$

(a) (i) Establish the condition of equilibrium of a particle on a rough plane curve. [5]

(ii) Let G and G_1 be the moments of the couple of a system of coplanar forces, when the system of forces is reduced with respect to two different bases O and O_1 respectively. Show that the moment of the couple, when the system is reduced with respect to the middle point of OO_1, is $\frac{1}{2}(G + G_1)$. [5]

(b) (i) Find the centre of gravity of a uniform arc of a circle making an angle 2α at the centre. [4]

(ii) If the density of a complete circular arc varies as the square of the distance from a point O on the arc, show that its centroid divides the diameter through O in the ratio **3:1.** [6]

(c) (i) Deduce the expressions for the radial and transverse components of velocity of a particle moving in a plane. [6]

(ii) If the angular velocity of a moving point about a fixed origin be constant, then show that the transverse acceleration varies as the radial velocity. [4]

(d) (i) Find the tangential and normal components of acceleration of a particle moving along a plane curve. [6]

(ii) If the tangential and normal accelerations of a particle moving in a plane curve be equal, then find the expression for the velocity. [4]

(e) (i) A particle moving with simple harmonic motion in a straight line has velocities v_1, v_2 at distances x_1, x_2 from the centre of path. Show that the period of motion is [5]

$$2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}.$$

(ii) Show that the work done in raising a number of particles each from one position to another is **Wh,** where **W** is the total weight of the particles and **h** is the distance through which the centre of gravity of the particles is raised. [5]

B.A./B.Sc. 5th Semester (General) Examination, 2021(CBCS)
Subject: Mathematics
Course: BMG5DSE1A3
(Linear Algebra)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) If U and W be two subspaces of a vector space V over a field F, then show that U∪W is a sub space of V if and only if either U⊂W or W⊂U. [5]

(b) (i) Is it possible to express the vector v=(2,−5,3) in $\mathbb{R}^3$ as a linear combination of the vectors u_1=(1,−3,2), u_2=(2,−4,−1), u_3=(1,−5,7)? Justify your answer. [3]

(ii) Let W={(a,b,c) ϵ $\mathbb{R}^3$: $a^2+b^2+c^2 \le 1$}. Show that W is not a subspace of $\mathbb{R}^3$. [2]

(c) (i) Find a basis and dimension of the subspace W={(a,b,c) ϵ $\mathbb{R}^3$: a+b+c=0}of $\mathbb{R}^3$. [2]

(ii) Prove that any maximum number of linearly independent vectors in S form a basis of V, where S spans the vector space V. [3]

(d) (i) Prove that kernel of F is a subspace of V and image of F is a subspace of U, where V, U are vector spaces over the same field of scalars and F: V→ U is a linear transformation. [3]

(ii) Prove that ker F={0} if and only if F is an onto mapping, where F:V→V is a linear [2] transformation and V is a finite dimensional vector space.

(e) (i) Find the matrix of F(x, y, z) = (3x+2y−4z, x−5y+3z) in the basis of S={(1,1,1), (1,1,0), (1,0,0)}, where $F:R^3 \to R^2$ is a linear transformation. [3]

(ii) Let $F: \mathbb{R}^3 \to \mathbb{R}^2$ defined by F(x, y, z) = (|x|, y+z). Is F linear? State reason. [2]

(f) Show that F is an isomorphism if and only if F is non-singular, where F: V→ U is a linear transformation and dimV = dimU. [5]

(g) (i) Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 4 \\ 6 & 4 & 5 \end{bmatrix}$. [2]

(ii) Find all eigen values and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$. [3]

(h) Find the dual basis of the basis S={(1,−1, 3), (0, 1, −1), (0, 3, −2)} of $\mathbb{R}^3$. [5]

2. Answer any three questions: 10×3 = 30

(a) (i) If A and P be both n×n matrices and P be non-singular, then show that A and $P^{-1}AP$ have the same eigen value. [5]

(ii) Prove that each eigen value of a real orthogonal matrix has unit modulus. [5]

(b) (i) Find a basis and the dimension of the image of A and the kernel of A, where $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$. [3+3]

(ii) Find the coordinates of an arbitrary vector v = (a,b,c) relative to the basis S = {(1, 1, 0), (0, 1, 1), (1, 2, 2)}. [4]

(c) (i) Let $\Phi_1 \Phi_2,\ldots,\Phi_n \in V^*$ be defined by $\Phi_i(v_j)=0$ for i≠j and $\Phi_i(v_j)=1$for i=j, where $\{v_1, v_2, \ldots, v_n\}$ is a basis of V over a field K. Then show that $\{\Phi_1, \Phi_2,\ldots,\Phi_n\}$ is a basis of V^*, where V^* is the dual space of V. [6]

(ii) Find the dimension of the subspace of $M_{n\times n}(\mathbb{R})$ consisting of all those matrices whose sum of each row is zero. [4]

(d) (i) Express v = (3, 7, −4) in $\mathbb{R}^3$ as a linear combination of the vectors u_1= (1, 2, 3), u_2 = (2,3,7), u_3 = (3,5,6). [4]

(ii) If S and T be two non-empty finite subsets of a vector space V over a field F and S⊆T. Then show that span(S)⊆span(T) and span[span(S)] = span(S). [3+3]

(e) (i) Determine whether the vectors (1, 1, 2), (1, 2, 5), (5, 3, 4) form a basis of $\mathbb{R}^3$or not? [3]

(ii) Prove that for every finite dimensional vector space V over a field, there exists a basis. [7]