B.A/B.Sc 5th Semester (General) Examination, 2021 (CBCS)

Subject: Mathematics

Course: BMG5SEC31

(Probability & Statistics)

Time: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

Answer any eight questions $8\times5 = 40$

(1) Show that the function $|x|$ in $(-1,1)$ and zero elsewhere is a possible probability density function, and find the corresponding distribution function. [5]

(2) Write down the probability mass function of binomial distribution. Let X be a Poisson μ variate. Find $Var(X)$. [1+4]

(3) (i) If $F(x)$ is a distribution function of a random variable X, prove that [3]

$$P(a \le x \le b) = F(b) - F(a-0).$$

(ii) Prove that $E(c) = c$ for any constant c. [2]

(4) Prove that the second order moment of a random variable X is minimum about the mean of X. [5]

(5) If X is a $\gamma(m)$ variate, then find the distribution of $\sqrt{X}$. [5]

(6) Let X be the value of points of appearing face when a die is rolled, Y, the value for rolling of second die and $Z = X + Y$. What are $E[X|Y = 5]$ and $E[Z|Y = 5]$? [3+2]

(7) Find the moment generating function of exponential distribution. [5]

(8) Let X and Y be two jointly continuous random variables with joint probability density function [3+2]

$$f(x,y) = cx^2y, 0 \le y \le x \le 1$$
$$= 0, \text{ elsewhere.}$$

Find the marginal density functions of X and Y and check whether they are independent or not.

(9) Prove that the distribution function is continuous to the right at every point \`$a$' but discontinuous to the left at every point \`a'. [5]

(10) Let X and Y be two jointly continuous random variables with joint probability density function [2+3]

$$f(x,y) = x + cy^2, 0 \le x \le 1, 0 \le y \le 1$$
$$= 0, \text{ elsewhere.}$$

Find the constant c.

Find $P\left(0 \le X \le \frac{1}{2}, 0 \le Y \le \frac{1}{2}\right)$.

B.A/B.Sc 5th Semester (General) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMG5SEC32
(Mathematical Finance)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

Answer any eight questions 8×5 = 40

(1) Explain the term hedging. A debt of Rs. 50,000 is to amortized over 7 years at 7% interest p. a. compounded annually. What value of monthly payment will achieve this? [2+3]

(2) Define duration of a fixed income security. Show that $\Delta P = -D_M P \Delta\lambda$, where P is the price of security, λ its yield and D_M the modified duration. [5]

(3) (i) Define swap and give an example of interest rate swap. [3]

(ii) Define maximum variants point. [2]

(4) Differentiate between systematic and non-systematic risk. Which of them can be reduced by diversification? [2+3]

(5) Mr. Hari buys European put option with a strike price of Rs. 100 per share to purchase 1000 shares of ABC company after 4 months. Option price is Rs. 5 per share. If the price of one share id Rs. 98 on expiration date, what will be Mr. Hari's gain/loss if the option is exercised? Should he exercise the option in this case? [5]

(6) Consider a non-dividend paying stock with current value Rs. 250. Assuming that the risk free rate is 6% p.a. compounding continuously, find the forward price and the initial value of a 1-year long forward contact on the stock. After six years, the price of the stock is Rs. 267 and the risk-free interest rate is still 6%. What is the forward price and what is the value of the above forward contact? [5]

(7) A major lottery advertises that it pays the winner Rs. 10 million. However this prize money is paid at the rate of Rs. 500,000 each year (with the first payment being immediate) for a total of twenty payments. What is the present value of this prize at 10% interest compounded annually? [5]

(8) Prove that the curve in an $\bar{r} - \sigma$ diagram defined by non-negative mixtures of two assets 1 and 2 lies within the triangular region defined by the two original assets and the point on the vertical axis of height $A = (\bar{r}_1\sigma_2 + \bar{r}_2\sigma_1)/(\sigma_1 + \sigma_2)$. [5]

(9) What will be the price of a 5% bond with face value Rs. 1000 and with annual coupon payments, three years to maturity and whose yield is zero? [5]

(10) Suppose there are three uncorrelated assets. The rates of return of each has variance 1, and mean values are 1, 2 and 3 respectively. Find the efficient portfolio are the minimum variance point. [5]

B.A/B.Sc 5th Semester (General) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMG5SEC33
(Mathematical Modeling)

Time: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

Answer any eight questions $8 \times 5 = 40$

(1) An m-lb weight is placed upon the lower end of a coil spring suspended from the ceiling and comes to rest in its equilibrium position, thereby stretching the spring l in. At time $t = 0$ the weight is then struck so as to set it into motion with an initial velocity of v_0 ft/sec, directed downward. The medium offers a resistance in pounds numerically equal to $a\frac{dx}{dt}$, where $\frac{dx}{dt}$ is the instantaneous velocity in feet per second and a is constant. Determine the resulting displacement of the weight as a function of time. [5]

(2) A 32-lb weight is attached to the lower end of a coil spring suspended from the ceiling. The weight comes to rest in its equilibrium position, thereby stretching the spring 2 ft. The weight is then pulled down 6 in. below its equilibrium position and released at $t = 0$. No external forces are present; but the resistance of the medium in pounds is numerically equal to $10\frac{dx}{dt}$,where $\frac{dx}{dt}$ is the instantaneous velocity in feet per second. Determine the resulting motion of the weight on the spring.

(3) A 64-lb weight is placed upon the lower end of a coil spring suspended from a rigid beam. The weight comes to rest in its equilibrium position, thereby stretching the spring 2 ft. The weight is then pulled down 1 ft below its equilibrium position and released from rest at t = 0. What is the position of the weight at $t = 5\pi/12$? How fast and which way is it moving at the time? [5]

(4) A 6-lb weight is hung on the lower end of a coil spring suspended from the ceiling. The weight comes to rest in its equilibrium position, thereby stretching the spring 4 in. Then beginning at $t = 0$ an external force given by $F(t) = 27 \sin At — 3 \cos At$ is applied to the system. If the medium offers a resistance in pounds numerically equal to three times the instantaneous velocity, measured in feet per second, find the displacement as a function of the time. [5]

(5) A 20-lb weight is attached to the lower end of a coil spring suspended from the ceiling. The weight comes to rest in its equilibrium position, thereby stretching the spring 6 in. Various external forces of the form $F(t) = \cos \omega t$ are applied to the system and it is found that the resonance frequency is 0.5 cycles/sec. Assuming that the resistance of the medium in pounds is numerically equal to $a\frac{dx}{dt}$, where $\frac{dx}{dt}$ is the instantaneous velocity in feet per second, determine the damping coefficient a. [5]

(6) A circuit has in series with a constant electromotive force of 100 V, a resistor of 10 Ω, and a capacitor of 2×10^{-4} farads. The switch is closed at time t = 0, and the charge on the capacitor at this instant is zero. Find the current at time $t > 0$. [5]

(7) Let $u(x,t)$ denote the temperature function in a rod of length L at any point x at time t. The sides of the rod are insulated and kept initially at temperature $\sin(\frac{\pi x}{L})$. Both ends of the rod are quickly cooled at 0° C and are kept at that temperature. Through appropriate modeling, obtain an expression for the temperature $u(x,t)$ of the string at any time t. [5]

(8) Find the traffic density $\rho(x,t)$ satisfying the traffic equation

$$\frac{\partial \rho}{\partial t} + (x \sin t)\frac{\partial \rho}{\partial x} = 0$$

with initial traffic density $\rho(x,0) = 1 + \frac{1}{1+x^2}$.

(9) Find the traffic density $\rho(x,t)$ satisfying the traffic equation [5]

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0$$

where u, the velocity of the car, is a function of traffic density alone and is given by $u(\rho) = u_{max}\left\{1 - \left(\frac{\rho}{\rho_{max}}\right)^2\right\}, 0 \leq \rho < \rho_{max}$

with initial traffic density as

$$\rho(x,0) = \begin{cases} 150, & x < 0 \\ 150\left(1 - \frac{x}{2}\right), & 0 \leq x < 1 \\ 80, & x \geq 1 \end{cases}$$

(10) A homogenous flexible string in a guitar is stretched between two fixed points (0, 0) and (2, 0), the length of the string being 2 units. The string of the guitar is initially plucked from rest from a position $\sin^3(\frac{\pi x}{2})$. Find the displacement $u(x,t)$ of the string of the guitar at time t. [5]