B.A./B.Sc. 3rd Semester (Honours) Examination, 2018 (CBCS)

Subject : Mathematics

(Numerical Methods)

Paper : BMH3CC07

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols bear usual meanings

Group-A

1. Answer *any five* questions: 2×5=10

(a) Derive Newton-Raphson formula for finding the m-th root of a given positive real number N in the following form:

$$x_{n+1} = \frac{(m-1)x_n^m + N}{m x_n^{m-1}} \quad (n = 0, 1, 2, \ldots)$$

(b) Find the value of $\Delta(x^2 + e^x + 2)$

(c) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}$ by Gauss-Jordan method.

(d) Given $y = x^3 + 3x^2 - x$. Compute the relative error in the value of y if $x = \sqrt{2}$ and $\sqrt{2} = 1{\cdot}414$.

(e) What do you mean by truncation error in numerical method? Give an example of it.

(f) What is meant by degree of precision of a quadrature formula? Illustrate why the degree of precision of Simpson's one-third formula is 3.

(g) Obtain the Trapezoidal formula for numerical integration for two points.

(h) Write Simpson's composite three-eighth rule for the evaluation of $\int_a^b f(x)dx$ stating the condition of sub-division of the interval $[a, b^-]$.

Group-B

2. Answer *any two* questions: 5×2=10

(a) (i) Prove that $\Delta^k y_0 = \sum_{i=0}^{k} (-1)^i \binom{k}{i} E^{k-i} y_0$

(ii) Show that divided difference depends upon scale but not on origin. 3+2=5

(b) (i) Show that the order of convergence of secant method is 1·618.

(ii) If $f(x)$ is a polynomial of degree 2 prove that $\int_0^1 f(x)dx = \frac{1}{12}[5f(0) + 8f(1) - f(2)]$

2+3=5

(c) Solve the following system of linear equations by LU-factorization method with the usual meaning of the symbols L and U.

$$2x - 6y + 8z = 24$$

$$5x + 4y - 3z = 2$$

$$3x + y + 2z = 16$$

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(d) Find, by the modified Euler's method, the value of y for $x = 0{\cdot}05$ from the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $y = 1$ when $x = 0$, correct up to four places of decimal.

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Group-C

3. Answer *any two* questions: 10×2=20

(a) (i) Find the function whose first difference is e^x taking the step size $k = 1$.

(ii) Prove that the sum of Lagrange's coefficients is unity.

(iii) Construct the difference table for $f(x) = x^3 + 2x + 1$ for $x = 0, 1, 2, 3, 4$. Comment on the differences of third order.

4+4+2=10

(b) (i) Discuss when a fourth order Runge-Kutta method for the solution of an initial value problem reduces to Simpson's one-third quadrature formula.

(ii) Deduce Simpson's $\frac{1}{3}$rd rule from the area underlying a parabola $y = ax^2 + bx + c$ bounded by the x-axis and the ordinates y_0 and y_2 where the parabola passes through the points $(-h, y_0)$, $(0, y_1)$ and (h, y_2).

(iii) Show that regula-falsi method converges linearly.

2+4+4=10

(c) (i) What do you mean by order of Convergence of an iterative method?

(ii) Derive Simpson's one-third quadrature composite rule by integrating Newton's forward difference interpolation formula.

(iii) If α and β are two real roots of the quadratic equation $ax^2 + bx + c = 0 (a \neq 0)$ show that the iteration method $x_{k+1} = -\frac{b}{x_k + a}$ is convergent near $x = \alpha$ if $|\alpha| < |\beta|$. 2+5+3=10

(d) (i) Compute the percentage error in $f(x)$ for $f(x) = 2x^3 - 4x$ at $x = 1$ when the error in x is 0·04.

(ii) Describe how Gauss-elimination method is modified in Gauss-Jordan method is solving a system of linear equations.

(iii) Deduce the condition of convergence of the fixed point iteration process. Justify the name 'fixed point'.

2+3+(4+1)=10

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