B.A./B.Sc. 3rd Semester (General) Examination, 2018 (CBCS)

Subject : Mathematics

Paper : BMG3SEC11

(Logic and Sets)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols and notations have their usual meaning.

Group–A

1. Answer *any five* questions: 2×5=10

(a) "I am a Liar"— Is it a statement or not? Justify your answer.

(b) Translate the statement — "Ramesh goes out for a walk if and only if it is not raining or the moon is out." into symbolic form.

(c) Find the negation of the statement — $\exists x p(x) \wedge \exists y q(y)$ where $p(x)$ and $q(y)$ are predicates on a set D.

(d) Determine whether the statement formula $(p \vee q) \wedge \sim (p \wedge q)$ is a tautology, a contradiction or a contingent.

(e) If S, T be the subsets of an universal set U, prove that S and T are disjoint if $S \cup T = S \Delta T$.

(f) Let $A = \{x \in z^+ : x \leq 30 \text{ and } x \text{ is a multiple of } 4\}$ and $B = \{x \in z^+ : x \leq 30 \text{ and } x \text{ is a multiple of } 6\}$. Then verify the counting principle for the sets A and B.

(g) Find the equivalence classes determined by the equivalence relation on $\mathbb{Z}$ defined by

$a \equiv b \pmod 3, a, b \in \mathbb{Z}$.

(h) For each $n \in Z^+$, let $A_n = [-2n, 3n]$. Determine $A_3 \Delta A_4$.

Group–B

2. Answer *any two* questions: 5×2=10

(a) (i) Verify by truth tables: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

(ii) Prove that if $\models$ A and $\models$ A → B, then $\models$ B. 2+3=5

(b) (i) Let $p(x)$ and $q(x)$ be two open statements defined for a specific universe and $\forall x[p(x) \rightarrow q(x)]$ be a true statement. Write its contrapositive and inverse statement.

(ii) Prove the De Morgan's law: $\sim\forall x(p(x)) \equiv \exists x \sim p(x)$, where $p(x)$ is the predicate on a set D. (1+1)+3=5

(c) (i) Define composition of two relations.

(ii) Draw Venn's diagram of the set $A \Delta B$ where A, B are the subsets of a universal set U.

(iii) Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C), A, B, C \subset U$. 1+1+3=5

(d) (i) Write the partitions of the set $A \cup B$, where A, B are subsets of a universal set.

(ii) In Z_{10}, which of the following equivalent classes are equal?
[2], [–5], [5], [–8], [12], [15], [–3], [7], [22].

(iii) Prove that the relation of congruence (module 5) is an equivalence relation. 1+1+3=5

Group–C

3. Answer *any two* questions: 10×2=20

(a) (i) Explain 'Universal quantifier' and 'existence quantifier'.

(ii) Find the truth table of the statement form $\left(\left((\sim A) \vee B\right) \Rightarrow C\right)$

(iii) Prove that $\mathcal{B}$ and $\mathcal{C}$ are logically equivalent *iff* $(\mathcal{B} \Leftrightarrow \mathcal{C})$ is a tautology. 2+4+4=10

(b) (i) Prove that intersection of two equivalence relations is an equivalence relation.

(ii) Show that the number of different reflexive relations on a set of n elements is 2^{n^2-n}. 5+5=10

(c) (i) Prove that $(A \cup B)' = A' \cap B'$. Where ' ' ' denotes the complement of a set.

(ii) Give an example of a relation which is not anti-symmetric.

(iii) Prove that the set of all even integers is countable. 4+2+4=10

(d) (i) Give an example to prove that $A \cap B = A \cap C$ does not imply $B = C$.

(ii) If $A\Delta B = A\Delta C$ then prove that $B = C$.

(iii) Let S be the set of all odd integers then find an empty relation on S.

(iv) Find the Domain of the relation $R = \{(a, 2), (b, 2), (c, 1), (a, 1), (d, 5)\}$. 3+4+2+1=10

B.A./B.Sc. 3rd Semester (General) Examination, 2018 (CBCS)

Subject : Mathematics

Paper : BMG3SEC12

(Analytical Geometry)

Time: 2 Hours **Full Marks: 40**

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Symbols and notations have their usual meaning.

Group–A

1. Answer *any five* questions: 2×5=10

(a) What does the following equation represent?
$7x^2 + 10xy + 4y^2 - 6x + 4y - 11 = 0$. Justify your answer.

(b) Find the eccentricity of the ellipse $\left(\frac{x-4}{1}\right)^2 + \left(\frac{y-5}{3}\right)^2 = 1$.

(c) Find the angle between the line $\frac{x-7}{1} = \frac{y+5}{-3} = \frac{z-2}{2}$ and the plane $3x + 4y = 1$.

(d) Find the shortest distance from the point (2, 1, –1) to the line $\frac{x}{2} = \frac{y}{3} = \frac{z-1}{2}$.

(e) For what value of *K*, the lines $\frac{x}{K} = \frac{y}{5} = \frac{z-1}{-1}$ and $\frac{x-6}{3} = \frac{y+1}{2} = \frac{z}{2K}$ will be perpendicular?

(f) Find the equation of the sphere which passes through the origin and intercepts *a*, *b*, *c* on the *x*, *y* and *z* axes respectively.

(g) Find the equation of the cylinder generated by the straight lines parallel to z-axis and passes through the curve of intersection of the plane $lx + my + nz = p$ and the surface $ax^2 + by^2 + cz^2 = 1$.

(h) The origin is shifted to the point (2, –3) without changing the direction of the axes. Find the co-ordinate of the point referred to the new set of axes if its co-ordinate w.r.t. the old set of axes be (3, 1).

Group–B

2. Answer *any two* questions: 5×2=10

(a) A plane passes through a fixed point (*a*, *b*, *c*) and cuts axes in *A*, *B*, *C*. Show that the centre of the sphere passing through the origin and *A*, *B*, *C* will lie on the surface $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$.

(b) Find the equation of the ellipse whose focus is the point (–1, 1), whose directrix is the straight line $x - y + 3 = 0$ and whose eccentricity is $\frac{1}{2}$.

(c) If (x_1, y_1) and (x_2, y_2) be the co-ordinates of the end points of a focal chord of the parabola $y^2 = 4ax$, then show that $x_1x_2 = a^2$ and $y_1y_2 + 4x_1x_2 = 0$.

(d) Find the centre and foci of the ellipse $x^2 + 4y^2 - 4x - 24y + 4 = 0$. 2+3=5

Group–C

3. Answer *any two* questions: 10×2=20

(a) (i) Prove that the tangent to an ellipse at a point makes equal angle with the focal radii from that point.

(ii) Find the equation of the right circular cylinder whose guiding curve is $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$. 5+5=10

(b) (i) Prove that the equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone if $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$.

(ii) Show that the plane $2x - y + 2z = 14$ touches the sphere $x^2 + y^2 + z^2 - 4x + 2y - 4 = 0$ and find the point of contact. 5+5=10

(c) (i) Find the radius of the circle formed by $x^2 + y^2 + z^2 - 6x + 2y + 14z - 10 = 0$, $2x + y = 1$.

(ii) Find the cone whose vertex is at (0, 0, 1) and lease is given by the curve $x^2 + y^2 + z^2 - 12x - 45 = 0, x + y = 2$. 4+6=10

B.A./B.Sc. 3rd Semester (General) Examination, 2018 (CBCS)

Subject : Mathematics

Paper : BMG3SEC13

(Integral Calculus)

Time: 2 Hours **Full Marks: 40**

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Candidates are required to give their answers in their own words as far as practicable.

Symbols and notation have their usual meaning.

Group–A

1. Answer *any five* questions: 2×5=10

(a) Find the reduction formula for $I_n = \int e^{-x}x^n dx$.

(b) Evaluate $\iint_R xy\left(x^2+y^2\right)dx\,dy$ over $R: [0, a\,; 0, b]$.

(c) Find the volume of the solid formed by the revolution of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a>b)$ about the major axis.

(d) If $\int \tan^4 x\, dx = p\tan^3 x + q\tan x + f(x) + c$, then find the value of p, q and $f(x)$.

(e) Show that $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ holds when $f(2a-x) = f(x)$.

(f) Find the value of $\int_0^{\frac{\pi}{2}} \cos^{2n+1} x\, dx$.

(g) If $g'(x) = f(x)$, find the value of $\int_a^b f(x)\, g(x)\, dx$.

(h) Find the length of the arc of the curve $y = \log\sec x$ from $x = 0$ to $= \frac{\pi}{3}$.

Group–B

2. Answer *any two* questions: 5×2=10

(a) (i) Evaluate : $\int_0^1\int_{y^2}^1\int_0^{1-x} x\,dz\,dx\,dy$

(ii) Show that $\int_{-2}^{2} \frac{x^2\sin x}{x^6+12}\,dx = 0$ 3+2=5

(b) (i) Find the area of the surface of revolution formed by the revolution of the parabola $y^2 = 4ax$ about x-axis and bounded by the section at $x = a$.

(ii) Evaluate : $\int \frac{2x+1}{x(x+1)} dx$. 3+2=5

(c) (i) Find the reduction formula for $\int \tan^n x \, dx$, where n is a positive integer greater than 1.

(ii) Evaluate : $\iint_R \sin(x+y)dxdy$, where $R : \left\{0 \le x \le \frac{\pi}{2}, 0 \le y \le \frac{\pi}{2}\right\}$. 3+2=5

(d) If $I_{m,n} = \int x^m(1+x)^n dx$, show that $(m+1)I_{m,n} = x^{m+1}(1+x)^n - nI_{m+1,n-1}$. 5

Group–C

3. Answer *any two* questions: 10×2=20

(a) (i) Evaluate : $\iiint (x+y+z+1)^4 \, dxdydz$ over the region defined by $x \ge 0, y \ge 0, z \ge 0$ and $x+y+z \le 1$.

(ii) Show that $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx = \frac{1}{\sqrt{2}} ln(\sqrt{2}+1)$.

(iii) Find the length of the arc of the curve $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$ between the points $x = 0$, $x = 3$. 4+4+2=10

(b) (i) Show that $\lim_{n\to\infty} \left[\frac{1^2}{n^2+1^2} + \frac{2^2}{n^2+2^2} + \cdots + \frac{1}{2}\right] = \frac{1}{3} log2$.

(ii) Evaluate : $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$

(iii) Find the length of one arch of the cycloid

$x = a(\theta - \sin\theta)$; $y = a(1 - \cos\theta)$ 3+4+3=10

(c) (i) Find the value of $\int_0^{\frac{\pi}{2}} \cos^3 x \cos 2x \, dx$.

(ii) If $I_{p,q} = \int_0^{\frac{\pi}{2}} \cos^p x \cos qx \, dx$, show that $I_{p,q} = \frac{p}{p+q} I_{p-1,\, q-1}$ and hence show that $\int_0^{\frac{\pi}{2}} \cos^n x \cos nx \, dx = \frac{\pi}{2^{n+1}}$.

(iii) Compute the value of $\iint_R y \, dx \, dy$ where R is the region defined by the first quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 3+4+3=10

(d) (i) Find the volume of the solid obtained by revolving the cardioide $r = a(1 + \cos\theta)$ about the initial line.

(ii) Evaluate : $\iint_R xy\left(x^2 + y^2\right)dx\,dy$ over $R[0, a\ ; 0, b]$.

(iii) Let $f(x, y)$ be defined over $R : \{0 \le x \le 1\ ;\ 0 \le y \le 1\}$ by

$f(x, y) = 1$ if x be irrational

$= 3y^2$ if x be rational

Show that $f(x, y)$ is not integrable over R. 4+3+3=10