B.A./B.Sc. 3rd Semester (Honours) Examination, 2022 (CBCS)

Subject : Mathematics

Course : CC-VII (BMH3CC07)

(Numerical Methods)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Given $= x_1x_2 + x_1x_3 + x_2x_3$, find error in the computation of u at $x_1 = 2{\cdot}104$, $x_2 = 1{\cdot}935$, $x_3 = 0{\cdot}845$.

(b) Compute $\sqrt{2}$ using the algorithm $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$, taking $x_0 = 1{\cdot}4$

(c) Find the number of significant figure in $X_A = 1{\cdot}8921$ given its relative error as $0{\cdot}1\times10^{-2}$.

(d) What do you mean by order of convergence of an iterative method?

(e) Show that Simpson's $^1/_3$ rd rule is exact for integrating a polynomial of degree 3.

(f) How do you interpret the statement Euler's method is a first order Range–Kutta method?

(g) What is meant by degree of precision of a quadrature formula? Illustrate why the degree of precision of Trapezoidel's rule is 1.

(h) Show that $\Delta \log f(x) = \log\left\{1 + \frac{\Delta f(x)}{f(x)}\right\}$.

2. Answer *any two* questions: 5×2=10

(a) Prove that the sum of Lagrange's co-efficients is unity.

(b) Prove that $\Delta^n f(x) = \sum_{i=0}^{n} (-1)^i \binom{n}{i} f(x+\overline{n-i}h)$.

Hence or otherwise deduce $\Delta^n y_0 = \sum_{i=0}^{n} (-1)^i \binom{n}{i} y_{n-i}$. 3+2=5

(c) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$

by Gauss–Jordan method.

(d) Solve the system of equations:

$x + 2y + 3z = 14$

$2x + 5y + 2z = 18$

$3x + y + 5z = 20$

by LU-decomposition method.

3. Answer *any two* questions: 10×2=20

(a) (i) Show that the 'remainder' in approximating $f(x)$ by the interpolation polynomial using distinct interpolating points $x_0, x_1, x_2, \ldots, x_n$ lying in $[a, b]$ is of the form $(x - x_0)(x - x_1)(x - x_2) \ldots (x - x_n)\frac{f^{(n+1)}(\xi)}{(n+1)!}$, where $\xi \in [a, b]$.

(ii) Explain the method of bisection for computing a simple real root of the equation $f(x) = 0$ and discuss the convergence of this iterative process. 5+(3+2)=10

(b) (i) Let $y = ax^2 + bx + c$ be the equation of the parabola passing through $(-h, y_0), (0, y_1)$ and (h, y_2). Find the area underlying the parabola bounded by the x-axis and two ordinates at $-h$ and h using Simpson's $^1/_3$ rd rule of integration. What conclusion do you draw from the result?

(ii) Show that Newton–Raphson method has a quadratic rate of convergence.

(iii) Obtain the relative error of $u = x_1^{m_1}.x_2^{m_2} \ldots x_n^{m_n}$ in terms of the relative errors of $x_1, x_2, \ldots x_n$. (3+1)+3+3=10

(c) (i) Describe modified Euler's method for solving the differential equation $\frac{dy}{dx} = f(x, y)$ in a finite interval $[a, b]$ assuming that $y(a)$ has a known value y_0. Give its geometrical interpretation.

(ii) Derive Newton–Cote's integration formula (error is not required) and deduce the particular formula with two sub-intervals. (4+2)+(3+1)=10

(d) (i) Find the greatest eigenvalue and the corresponding eigne-vector of the matrix

$$A = \begin{pmatrix} 4 & 0 & 2 \\ 0 & -1 & 0 \\ 2 & 0 & 4 \end{pmatrix}$$

by Power method.

(ii) Describe Gauss's elimination method for numerical solution of a system of linear equations. Explain in this method, pivoting process involved. 5+(3+2)=10