B.A./B.Sc. 1st Semester (Honours) Examination, 2017 (CBCS)

Subject : Mathematics

Paper : BMHICC-I

Time: 3 Hours **Full Marks: 60**

The figures in the right hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meaning.

1. Answer *any ten* questions from the following: 2×10=20

(a) Find the points of inflexion on the curve $y = (\log x)^3$.

(b) Using L'Hospital's rule, evaluate $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$.

(c) If $I_n=\int(\log x)^n dx$, show that $I_n=x(\log x)^n - n\, I_{n-1}$ for any positive integer n. Hence evaluate $\int(\log x)^2\, dx$.

(d) The circle $x^2 + y^2 = a^2$ revolves round the X-axis. Find the surface area of the whole surface generated.

(e) Show that for the equation $ax^2 + 2hxy + by^2 = 0$, $ab - h^2$ is an invariant under a rotation of rectangular axes.

(f) Find the centre and radius of the circle represented by $x^2 + y^2 + z^2 = 1$, $x + y = z$.

(g) Solve $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$.

(h) Solve $y = px + p - p^2$ for complete and singular solution, where $p = \frac{dy}{dx}$

(i) Find $\frac{d^9}{dx^9}\left(x^3\log_e(3x)\right)$.

(j) Find the envelope of the family of circles $x^2+(y-a)^2=25$, a being the parameter.

(k) Find the parametric equations of

(i) the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$

(ii) the hyperbola $25(x-1)^2-9(y+2)^2 = 225$.

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(l) Evaluate $\frac{1}{2}\int_0^2 \sqrt{4+x^2}\,dx$ and interpret it geometrically.

(m) Find the equation of the sphere whose end points of a diameter are (1, 0, –3) and (–2, 7, 5). Also examine that whether it cuts the X-axis or not.

(n) Find the equation of a right circular cylinder whose axis is the Y-axis and which has radius 5 units.

(o) Find the differential equation of all circles, which pass through the origin and whose centres are on the Y-axis.

2. Answer *any four* questions from the following: 5×4=20

(a) State and prove Leibnitz's theorem. 1+4=5

(b) Find the asymptotes of the curve

$x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0.$ 5

(c) Obtain a reduction formula for $\int \sec^n x\,dx$, n being a positive integer. Hence find the value of $\int \sec^6 x\,dx$.

3+2=5

(d) Find the locus of the points of intersection of perpendicular generators of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$. 5

(e) A sphere of radius k passes through the origin and meets the axes in A, B, C. Prove that the locus of the centroid of the triangle ABC is the sphere $9(x^2 + y^2 + z^2) = 4k^2$. 5

(f) Solve *any one* of the following: 5×1=5

(i) $y\,(xy + 2x^2y^2)\,dx + x\,(xy - x^2y^2)\,dy = 0$

(ii) $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x^2}(\log y)^2$

3. Answer *any two* of the following: 10×2=20

(a) (i) Show that the points of inflexion of the curve $y^2 = (x - a)^2(x - b)$ lie on the line $3x + a = 4b$.

(ii) Evaluate: $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x}$

(iii) Find the envelope of the family $\frac{x}{a}+\frac{y}{b}=1$, where the parameters a, b are connected by the relation $a^5 + b^5 = 1$. 3+3+4=10

(b) (i) Find the surface area of the solid generated by revolving the cycloid $x = a\,(\theta - \sin\theta)$, $y = a\,(1 - \cos\theta)$ about X-axis.

(ii) Derive a reduction formula for $\int \sin^m x \cos^n x\,dx$, where m, n are positive integers, greater than 1 and hence evaluate $\int \sin^4 x \cos^2 x\,dx$. 5+(3+2)=10

(c) (i) If a conic $\frac{l}{r} = 1 + e\cos\theta$ is cut by a circle passing through the pole in four points with radius vectors r_1, r_2, r_3, r_4, then prove that $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\frac{1}{r_4}=\frac{2}{l}$.

(ii) Reduce the following equation to the canonical form and determine the type of the quadric represented by it: 4+(5+1)=10

$3x^2 + 5y^2 + 3z^2 + 2yz + 2zx + 2xy - 4x - 8z + 5 = 0$

(d) (i) Find the particular solution of $\cos y\,dx + \left(1+2e^{-x}\right)\sin y\,dy = 0$, when $x = 0$, $y = \frac{1}{4}\pi$.

(ii) Solve the differential equation $(px - y)(x - py) = 2p$, by using the transformation $x^2 = u$ and $y^2 = v$, where $p = \frac{dy}{dx}$. 5+5=10
