B.A/B.Sc 6th Semester (Honours) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH6DSE31
(Mathematical Modelling)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions from the following: 6×5=30

(a) Explain the concept of Lyapunov stability of the equilibrium state of the differential equation $\frac{dx}{dt} = f(x)$. 5

(b) Investigate the asymptotic stability of the equilibrium points of the model equation $\frac{dx}{dt} = rx(1 - x/k)$. 5

(c) Derive the steady state difference equations for the queueing model $(M/M/1):(\infty/FCFS/\infty)$. 5

(d) A population is governed by the equation $\frac{dx}{dt} = x(e^{3-x} - 1)$. Find all equilibria. 5

(e) A drug is administered every six hours. Let $D(n)$ be the amount of the drug in the blood system at nth interval. The body eliminates a certain fraction p of the drug during each time interval. If the initial blood administered is D_0, find $D(n)$ and $\lim_{n\to\infty} D(n)$. 5

(f) Show that the equilibrium (x^*, y^*) with $x^* > 0, y^* > 0$ of the predator-prey model

$$\frac{dx}{dt} = x(1 - x/k) - \frac{axy}{x+A}$$
$$\frac{dy}{dt} = y\left(\frac{ax}{x+A} - \frac{aB}{A+B}\right)$$

is unstable if $k > A + 2B$ and asymptotically stable if $B < k < A + 2B$. 5

(g) Discuss generalised least squares estimator. 5

(h) The growth of a population satisfies the following difference equation

$$x_{n+1} = \frac{kx_n}{b + x_n}, \; b, k > 0 \; .$$

Find the steady state (if any). If so, is that stable? 5

2. Answer any three questions from the following: 3×10=30

(a) (i) Discuss the method of maximum likelihood.

(ii) In a railway yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes, calculate

(A) the average number of trains in the system.

(B) the average number of trains in the queue.

(C) the expected waiting time in the queue.

(D) the probability that the number of trains in the system exceeds 10. 6+4

(b) Discuss Malthus model equation of population growth $\frac{dN}{dt} = rN.$ Interpret the equation when the sign of r is reversed. 10

(c) Obtain the maximum likelihood estimator of σ^2 where μ (known) and σ are mean and standard deviation of a normal population respectively. Show that this estimator is unbiased. 10

(d) Show that if arrivals in a queue are completely random, then the probability distribution of the number of arrivals in a fixed time interval follows Poisson distribution. 10

(e) (i) Explain the modelling of a system in discrete time.

(ii) Derive the mathematical model of traffic flows. 4+6

B.A/B.Sc 6th Semester (Honours) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH6DSE32
(Industrial Mathematics)

Time: 3 Hours — Full Marks: 60

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Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions from the following: 6×5=30

(a) What do you mean by Hounsfield unit of a tissue? What is the range of a typical clinical CT scan between air and bone? 3+2

(b) Define Radon transform of a function with arguments t and θ. Explain with an example. 3+2

(c) Define unfiltered back projection for a function with arguments t and θ. 5

(d) If a continuous function f is equal to zero outside some disc and F is an integrable function of t and θ, then show that $\langle \mathcal{R}f, F\rangle = \langle f, \mathcal{R}^*F\rangle$, where $\mathcal{R}$ and $\mathcal{R}^*$ be the radon transformation and its adjoint respectively. 5

(e) Draw a curve showing the intensity of transmitted x-rays as a function of time following Beers law. Interpret the curve physically. 3+2

(f) Derive total spectral attenuation in terms of photoelectric absorption term, Compton scattering term, atomic number of the absorber, scattering attenuation constant and photoelectric constant. 5

(g) Write and explain the algorithm of CT scan. 3+2

(h) How can data be derived from a century old mummy using CT scan, keeping it intact? Write down the associated mathematical steps. 3+2

2. Answer any three questions from the following: 3×10=30

(a) Define p-wave, s-wave and Rayleigh wave kernels. Write a brief note on Optimal observables for multi-parameter seismic tomography. 10

(b) Show that the systematic rotation ω for tomographic experiments can be defined as

$$\begin{pmatrix}\omega_x\\ \omega_y\\ \omega_z\end{pmatrix} = \frac{1}{2}\begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix} \times \begin{pmatrix}u_x\\ u_y\\ u_z\end{pmatrix} = \frac{1}{2}\begin{pmatrix}\partial_y u_z - \partial_z u_y\\ \partial_z u_x - \partial_x u_z\\ \partial_x u_y - \partial_y u_x\end{pmatrix}.$$

Hence define tilt. What are its physical significances? 10

(c) Describe the evaluation of tomographic brain reconstruction techniques using shepp-logan phantom imaging. 10

(d) Set an example of back projection. Explain with graphs. 10

(e) Establish the CT reconstruction formula. 10

B.A/B.Sc 6th Semester (Honours) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH6 DSE33
(Group Theory-II)

Time: 3 Hours — Full Marks: 60

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Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions from the following: 6x5=30

(a) Show that S_3 cannot be written as internal direct product of two non-trivial subgroups. 5

(b) Prove that a group of order 45 is an Abelian group. 5

(c) Find the three 2-sylow subgroups of S_3. 5

(d) Let G be a group of order 187. Prove that any subgroup of order 17 in G must be normal. 5

(e) Let G be a group which acts on a set X. For $x \in X$, define stabilizer G_x of x and show that it forms a subgroup of G. 5

(f) Prove that the centre of S_n is trivial for n>2. 5

(g) Prove that any group of order 15 is cyclic. 5

(h) Let $O(2)$ be the group of 2×2 real orthogonal matrices and $\mathbb{R}^2$ be the Euclidean plane. Define a group action on the plane $\mathbb{R}^2$ and find its orbit. 5

2. Answer any three questions from the following: 3x10=30

(a) (i) Let G be a group of order pq where p and q are distinct primes $p > q$ and q does not divide $p - 1$. Then prove that G is cyclic.
(ii) Let G be a group and f be an automorphism of G. Show that the set $\{a \in G : f(a) = a\}$ forms a subgroup of G. 7+3

(b) (i) Let G be a finite group and $a \in G$ be such that $o(a) > 1$. Suppose that G has exactly two conjugacy classes. Prove that $|G| = 2$.
(ii) Let G be a group of order $p^\alpha m$ where p is a prime, α and m are positive integers, and p and m are relatively prime. Prove that G has a subgroup of order p^α. 5+5

(c) (i) Let H and K be two subgroups of a group G. Show that $Z(H \times K) = Z(H) \times Z(K)$.
(ii) Prove that for any group G, the set $\{a \in G : f(a) = a, \forall f \in Aut(G)\}$ forms a normal subgroup of G. 6+4

(d) (i) Prove that the direct product of two finite cyclic groups of order m and n is again a cyclic group if and only if $g.c.d(m,n) = 1$.
(ii) Prove that $Aut(\mathbb{Z}_2 \times \mathbb{Z}_3) = Aut(\mathbb{Z}_2) \times Aut(\mathbb{Z}_3)$. 6+4

(e) (i) Let H be a subgroup of a finite group G and suppose H acts on G under $*: H \times G \to G$ such that $*(h,g) = hg$. Prove that $o(H)|o(G)$.
(ii) Prove that there are no simple groups of order 63. 5+5