B.A./B.Sc. 6th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH6DSE31

(Mathematical Modelling)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) What are the limitations of mathematical modelling?

(b) Write down the assumptions of queueing model $(M/M/1):(N/FCFS/\infty)$.

(c) Find the average length of non-empty queue of a system $(M/M/1):(\infty/FCFS/\infty)$.

(d) Write down the relations between

(i) L_s and L_q

(ii) W_s and W_q of $(M/M/1):(\infty/FCFS/\infty)$

(e) What do you mean by service discipline of a queueing system?

(f) What is Allee effect?

(g) What is Malthns model?

(h) Define the Lotka-Volterra model for prey-predator system.

(i) Give an example of two species competition model.

(j) Define equilibrium point of a system.

(k) Give an example of a discrete prey-predator model.

(l) Find the equilibrium point of $\frac{dx}{dt} = x(x-1)$.

(m) Write down the logistic model of population growth explaining the different terms involved in it.

(n) What are the state variables for the dynamical models of ecosystem?

(o) What are the basic postulates for developing continuous time models of single species population?

2. Answer *any four* questions: 5×4=20

(a) A population satisfies the growth equation $x_{n+2} - 2x_{n+1} + 3x_n = 0$. Find the population in n-th generation. Also, find the steady state.

(b) Write down the logistic model for a single-species population. Hence, explain the concepts of carrying capacity and intra-species competition. 2+3

(c) Discuss density dependent growth model.

(d) Find the non-negative equilibrium of a population governed by $x_{n+1} = \frac{2x_n^2}{x_n^2+2}$ and investigate the stability. 3+2

(e) If the arrival process in a queueing system follows the poisson distribution then show that the associated random variable defined as inter-arrival time follows the exponential distribution.

(f) Discuss different states of a queueing system.

3. Answer *any two* questions: 10×2=20

(a) Obtain the maximum likelihood estimator of σ^2 where μ (known) and σ are mean and standard deviation of a normal population respectively. Show that this estimator is unbiased. 8+2

(b) In a railway yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes, calculate 2×5

(i) the average number of trains in the system.

(ii) the average number of trains in the queue.

(iii) the expected waiting time in the system.

(iv) the expected waiting time in the queue.

(v) the probability that the number of trains in the system exceeds 10.

(c) Consider the prey-predator system

$\frac{dx}{dt} = x(1 - x - y)$

$\frac{dy}{dt} = \beta(x - \alpha)y$, α, β being constants.

Investigate the nature of equilibrium points of the system.

(d) Define a cooperative system and give an example. Prove that the orbit of a system that is cooperative either converge to equilibrium or diverge to infinity.

B.A./B.Sc. 6th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH6DSE32

(Industrial Mathematics)

Time: 3 Hours **Full Marks: 60**

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Candidates are required to give their answers in their own words as far as practicable.

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1. Answer *any ten* questions: 2×10=20

(a) Write brief note on seismic image.

(b) Why do we require attenuated Radon transform?

(c) State one result of Fourier Slice theorem.

(d) Find the inverse Fourier transform of a step wave.

(e) Define band limited function. Where is it used?

(f) Plot the Fourier transformation of $f(x) = \sin(ax), |a| > 0$.

(g) What is the use of a space of bounded measurable functions?

(h) What are the applications of Sheep-Logan filter?

(i) Find the Radon transformation of the body given by

$$E(x,y) = \begin{cases} 1, & 1 \le x^2 + y^2 \le 9 \\ -1, & everywhere\ else. \end{cases}$$

(j) Why do we need to include the idea of Affine space? Explain with examples.

(k) What do you mean by low pass cosine filter?

(l) Where do you need point spread function?

(m) Which kind of tomography is important to study back projections?

(n) Find the area under the curve of Dirac delta function.

(o) State an application X-ray tomography.

2. Answer *any four* questions: 5×4=20

(a) Define the characteristic function of the interval $[-l, l]$. Find the Fourier sine transformation of this function. 2+3

(b) What is convolution back projection? Write down the optoacoustic tomography for this back projection. 2+3

(c) State and prove Nyquist's theorem.

(d) Applying the scaling property of the ideal ramp filter, find out fan-beam reconstruction formula.

(e) Evaluate the Fourier transform of weighted projections for each parametric change.

(f) Apply the Rayleigh-Plancherel theorem to the function $f(x) = e^{-|x|}$ in order to evaluate the integral

$$\int_{-\infty}^{\infty} \frac{1}{(1+\omega^2)^2}\, d\omega .$$

3. Answer *any two* questions: 10×2=20

(a) (i) Give an example of two random variables that are individually Gaussian distributed but their joint distribution is not Gaussian. Give proper justification.

(ii) If $A(\omega) = |\omega|.\left(\sin\frac{\left(\frac{\pi\omega}{2L}\right)}{\frac{\pi\omega}{2L}}\right).\, \Pi_L(\omega)$, then prove that

$$(\mathcal{F}^{-1}A)\left(\frac{n\pi}{L}\right) = \frac{4L^2}{\pi^3(1-4n^2)}.$$ 5+5

(b) (i) Show that the rotation vector ω can be defined as

$$\begin{pmatrix}\omega_x\\ \omega_y\\ \omega_z\end{pmatrix} = \frac{1}{2}\begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix} \times \begin{pmatrix}u_x\\ u_y\\ u_z\end{pmatrix} = \frac{1}{2}\begin{pmatrix}\partial_y u_z & - & \partial_z u_y\\ \partial_z u_x & - & \partial_x u_z\\ \partial_x u_y & - & \partial_y u_x\end{pmatrix}.$$

Hence define tilt. What are its physical significances?

(ii) Write and explain the algorithm of CT scan. How can data be derived from a century old mummy using CT scan, keeping it intact? Answer briefly with help of mathematical steps. 5+(2+3)

(c) (i) Show that the functions $F_1(t) = e^{(\alpha+i\omega)t}$ and $F_2(t) = e^{(\alpha-i\omega)t}$, where α and ω are real constants, satisfy the (second-order linear) differential equation

$$y'' - 2\alpha y' + (\alpha^2 + \omega^2)y = 0.$$

Using Euler's formula, show that the functions $y_1 = e^{\alpha t}\cos(\omega t)$ and $y_2 = e^{\alpha t}\sin(\omega t)$, also satisfy the same differential equation.

(ii) Define f by $f(x,y) = \begin{cases} x, x^2+y^2 \le 1 \\ 0, x^2+y^2 > 1 \end{cases}$. Compute the Radon transformation of $f\left(\frac{1}{2}, \frac{\pi}{6}\right)$.

5+5

(d) (i) Suppose that a Hanning window is applied to the ramp filter, using Fourier transform properties, find analytically the impulse responses.

(ii) What is Hounsfield unit of a tissue? Explain with plots. 5+5

B.A./B.Sc. 6th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH6DSE33

(Group Theory II)

Time: 3 Hours **Full Marks: 60**

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Group-A

1. Answer *any ten* questions: 2×10=20

(a) Show that the set of automorphisms of a group forms a group under the operation of composition of functions.

(b) Let $f: G \to G$ be a mapping defined by $f(x) = x^{-1}, \forall x \in$ G. Show that f is an automorphism if G is abelian.

(c) Let G be a group and $g \in G$. Show that the mapping $\phi_g(x) = gxg^{-1}$ for all $x \in G$ is an automorphism.

(d) Prove that a commutative group of order 10 is cyclic.

(e) Show that the direct product $\mathbb{Z} \times \mathbb{Z}$ is not a cyclic group.

(f) Show that the direct product $S_3 \times \mathbb{Z}$ of the groups S_3 and $\mathbb{Z}$ is an infinite non-commutative group.

(g) Let G_1, G_2 be two commutative groups. Show that direct product $G_1 \times G_2$ is a commutative group.

(h) State fundamental theorem for finite abelian groups.

(i) Find all sylow 2-subgroups of A_4.

(j) Show that any group of order p^2 is commutative, where p is a prime.

(k) Prove that no group of order 8 is simple.

(l) State Sylow's first theorem.

(m) Prove that a group of order 99 has a unique normal subgroup of order 11.

(n) Prove that a group G is commutative if and only if $G' = \{e\}$.

(o) Write the class equation of S_3.

Group-B

2. Answer *any four* questions: 5×4=20

(a) If G is an infinite cyclic group, then prove that $Aut(G)$ is a group of order 2.

(b) Show that every characteristic subgroup of a group G is a normal subgroup of G. Is the converse true? Support your answer. 3+2

(c) Show that the derived subgroup G' of a group G is a normal subgroup of G and $G/_{G'}$ is commutative. 3+2

(d) State and prove Cauchy's theorem for finite group. 1+4

(e) Prove that any group of order 30 is not simple.

(f) Show that every group of order 255 is cyclic.

Group-C

3. Answer *any two* questions: 10×2=20

(a) (i) Let G be a group and $Z(G)$ be the centre of the group G. Then show that $Inn(G)$ is isomorphic to the quotient group $G/_{Z(G)}$.

(ii) Find the number of inner automorphisms of the group S_3.

(iii) Show that $Aut(\mathbb{Z}_n) \simeq U_n$ 4+3+3

(b) (i) Let H and K be two finite cyclic groups of order m and n respectively. Prove that the direct product $H \times K$ is a cyclic group if and only if $gcd(m,n) = 1$.

(ii) Find the number of elements of order 5 in $\mathbb{Z}_{15} \times \mathbb{Z}_5$.

(iii) Let G be a finite p-group with $|G| > 1$. Prove that $|Z(G)| > 1$. 4+3+3

(c) (i) Let p be an odd prime. If G is a group of order $2p$, then show that either $G \cong \mathbb{Z}_{2p}$ or $G \cong D_p$.

(ii) Show that every group of order 99 is abelian.

(iii) Show that every cyclic group is abelian. 4+3+3

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