B.A/B.Sc 5th Semester (Honours) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH5CC11 (Partial Differential Equations and Applications)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) Prove that the general solution of the semilinear partial differential equation $Pp+Qq=R$ is $F(u,v)=0$ where u and v are such that $u=u(x,y,z)=c_1$ and $v=v(x,y,z)=c_2$ are solutions of $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$ [c_1, c_2 are constants]. [5]

(b) By the method of characteristics, solve the Cauchy problem: $pz+q=1$ with initial data $y=x, z=x/2$. [5]

(c) (i) Find the partial differential equation of all planes which are at a constant distance 'a' from the origin. [3]

(ii) Explain the concept of Cauchy problem for second order partial differential equation. [2]

(d) Derive the characteristic equations of the partial differential equation, $F(x,y,z,p,q)=0$. [5]

(e) (i) When is a second order linear partial differential equation in two independent variables classified into hyperbolic, parabolic and elliptic? [3]

(ii) Determine the region where the given partial differential equation $yu_{xx}-xu_{yy}=0$ is hyperbolic in nature. [2]

(f) Consider partial differential equation of the form $ar+bs+ct+f(x,y,z,p,q)=0$ in usual notation, where a, b, c are constants. Show how the equation can be transformed into its canonical form where $b^2-4ac<0$. [5]

(g) Obtain the solution of the diffusion equation $u_t=Ku_{xx}, K>0$, in the region $0<x<\pi$, $t>0$ subject to the conditions: [5]

i) $u(x,y)$ remains finite as $t\to\infty$.

ii) $u=0$ at $x=0$ and π for $t>0$.

iii) at $t=0$, $u(x,t)=x$ when $0\le x\le \pi/2$, $u(x,t)=\pi-x$ when $\pi/2<x\le\pi$.

(h) Solve: $(x^2-yz)p+(y^2-zx)q=z^2-xy$. [5]

2. Answer any three questions: 3×10 = 30

(a) (i) Using the transformation $\alpha=\ln x$, $\beta=\ln y$, transform the equation $x^2r-y^2t+xp-yq=\ln x$ to ordinary differential equations. [6]

(ii) Determine the characteristics strips of the equation $z=p^2-3q^2$ and obtain the integral [4] surface which passes through the curve $x=t,\ y=0,\ z=t^2$.

(b) (i) Reduce the partial differential equation $z_{xx}+2z_{xy}+z_{yy}=0$ to its canonical form. [6]

(ii) Form the partial differential equation by eliminating f from the given relation: $u=f(x^2+2yz, y^2+2zx)$. [4]

(c) Solve: $z_{xx}-2z_x+z_y=0$ by the method of separation of variables. Hence find the solution, when $z(0,y)=0$ and $z_x(0,y)=e^{-3y}$. [6+4]

(d) (i) A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $\sin^3 \pi x/l$. Find the displacement $u(x,t)$. [6]

(ii) Solve by the method of separation of variables $u_x=4u_y$, given that $u(0,y)=8e^{-3y}$. [4]

(e) (i) Prove that the solution of the initial value problem, $u_{xx}-u_{yy}=0,\ |x|<\infty,\ y>0$, $u(x,0)=f(x), u_y(x,0)=g(x)$ is $u(x,y)=\frac{1}{2}\left[f(x+y)+f(x-y)\right]+\frac{1}{2}\int_{x-y}^{x+y} g(t)\,dt$.. [6]

(ii) Show that the equation $x^2z_{xx}-y^2z_{yy}=0$ is hyperbolic in nature everywhere in the xy-plane. Find its characteristics. [4]