B.A./B.Sc. 5th Semester (Honours) Examination, 2022 (CBCS)

Subject : Mathematics

Course : BMH5DSE21

(Probability and Statistics)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions from the following: 2×10=20

(a) Show that for any three events A, B, and C of a random experiment, $P(ABC) \geq P(A) + P(B) + P(C) - 2$.

(b) The radius X of a circle has uniform distribution in (1,2). Find the variance of the area of the circle.

(c) Find the constant k for which $F(x) = \int_{-\infty}^{x} ke^{-|t|}dt$ is a probability distribution function.

(d) For a Poisson distribution, $P(X = 0) = P(X = 1)$. Find $P(X > 0)$.

(e) Let X be a Binomial (n, p)-variate. For what value of p, $\text{Var}(X)$ is maximum?

(f) If X is a normal (m, σ) variate, prove that $P(|X - m| > a\sigma) = 2[1 - \Phi(x)]$, where $\Phi(x)$ is a standard normal distribution function.

(g) Find the distribution of $Y = \sin X$, if X is distributed with probability density function $f(x) = \frac{1}{2}\cos x; x \epsilon \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(h) Find the moment generating function of the Poisson distribution.

(i) Define the characteristic function of a continuous random variable X.

(j) State central limit theorem in case of equal components.

(k) If the random variables X and Y have the same standard deviation, show that $U = X + Y$ and $V = X - Y$ are uncorrelated.

(l) Determine the correlation coefficient of the random variables X and Y if $\text{Var}(X) = 4$, $\text{Var}(Y) = 2$ and $\text{Var}(X + 2Y) = 15$.

(m) Show that the sample mean is consistent estimator of population mean.

(n) Write down the maximum likelihood function for the normal (m, σ) population.

(o) The random variable X is normal $(50, 20)$. Find $P(|X - 50| \leq 20)$ given that $\frac{1}{2\pi}\int_{-\infty}^{1} e^{-x^2/2}\,dx = 0{\cdot}8413$.

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2. Answer *any four* questions from the following: 5×4=20

(a) Define distribution function of a random variable. Show that the distribution function of a random variable X is continuous at all points. 1+4=5

(b) The height of adult people in West Bengal is normally distributed with parameters m and σ. In a gathering of 1000 people, how many of them have their heights more than 175 cm? Given that $m = 165$, $\sigma = 15$ and $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{2/3} e^{-t^2/2}\, dt = 0{\cdot}7475$.

(c) Prove that $P(a<X\leq b,\ c<Y\leq d)=F(b,\ d) + F(a,\ c) - F(b,\ c) - F(a,\ d))$ when $F(x,y)$ is a two dimensional distribution function of X and Y.

(d) If X_1, X_2 are independent random variables each having the density function given by $f(x) = 2xe^{-x^2}$ $0 < x < \infty$. Find the probability density function of the random variable $\sqrt{X_1^2 + X_2^2}$.

(e) If $(x_1, x_2, \ldots, x_n)$ be a simple random sample drawn from a normal $(\mu, 1)$ population, then show that $\left(\bar{x}^2 - \frac{1}{n}\right)$ is an unbiased estimator of μ^2.

(f) The joint density function of the random variables X and Y is given by $f(x,y) = x + y$, $0 < x < 1$, $0 < y < 1$ and zero, elsewhere. Find the distribution of XY.

3. Answer *any two* questions from the following: 10×2=20

(a) (i) State and prove Tchebycheff's inequality.

(ii) A random variable X has probability density function $12x^2(1-x)$, $0 < x < 1$ and zero, elsewhere. Compute $P(|X - m| \geq 2\sigma)$ and compare it with the limit given by Tchebycheff's inequality (m and σ^2 are mean and variance of X respectively. 5+5

(b) (i) The joint probability density function of (X, Y) is given by

$$
\begin{aligned}
f(x,y) &= e^{-x-y} \quad ; x \geq 0, y \geq 0 \\
&= 0 \quad ; \quad \text{elsewhere}
\end{aligned}
$$

Find the distribution function $F(x,y)$. Also find marginal probability density function of X.

(ii) Define moment generating function of Bivariate continuous random variable (X, Y). If the bivariate random variable (X, Y) follows normal distribution BN $(0, 0, 1, 1, \rho)$, then show that its moment generating function is given by

$M_{XY}(t_1, t_2) = e^{\frac{1}{2}(t_1^2 + t_2^2 + 2\rho t_1 t_2)}$. (2+2)+(1+5)

(c) (i) If for any pair of correlated random variables X and Y,
$U = X\cos\theta + Y\sin\theta$, $V = -X\sin\theta + Y\cos\theta$ (θ being constant), then show that U and V will be uncorrelated if θ is given by

$$\tan 2\theta = \frac{2\rho\sigma_x\sigma_y}{\sigma_x^2-\sigma_y^2}.$$

(ii) Obtain Bernoulli's theorem as a particular case of law of large numbers for equal components. 5+5

(d) (i) Find $100(1-\alpha)\%$ confidence interval of population mean μ when σ is unknown, if samples are drawn from Normal (μ, σ^2) population.

(ii) A sample of size n is drawn from Normal (μ, σ^2) population, the standard deviation σ being known. Discuss how you could test the hypothesis. 4+6

$H_0 : \mu = \mu_0$ against an alternative hypothesis

$H_1 : \mu = \mu_1 > \mu_0$ at significance level α.

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B.A./B.Sc. 5th Semester (Honours) Examination, 2022 (CBCS)

Subject : Mathematics

Course : BMH5DSE22

(Portfolio Optimization)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) What is beta?

(b) Why is standard deviation commonly employed as a measure of risk?

(c) Define the return relative.

(d) Define geometric mean return.

(e) What do you mean by CAMP model?

(f) Define measure of risk.

(g) What are the criticisms of variance as a measure of risk?

(h) Compare the rate of return ratios return on assets.

(i) Which fundamental factors derive beta?

(j) What do you mean by covariance of two random variables?

(k) State relationship between covariance and correlation.

(l) What is an efficient portfolio?

(m) What do you mean by risk of a portfolio?

(n) Discuss the common misunderstanding about the relationship between asset allocation and portfolio performance.

(o) How are fundamental betas superior to historical betas?

2. Answer *any four* questions: 5×4=20

(a) Derive the relationship between risk and return for efficient portfolios.

(b) Derive Marshal Blumis formula.

(c) Discuss the problems and issues faced in financial statement analysis.

(d) Explain the single index model proposed by William Sharpe.

(e) How does the efficient frontier change, when the possibility of lending and borrowing at a risk-free rate is introduced?

(f) Compare the following rate of return ratios:
Earning power, Return on capital employed and Return equity

3. Answer *any two* questions: 10×2=20

(a) The rates of return on stock X and market portfolio for 15 periods are given below:

Period	Return on stock X (%)	Return of market portfolio (%)
1	24	12
2	13	14
3	17	13
4	15	10
5	14	9
6	18	13
7	16	14
8	6	7
9	– 8	1
10	13	12
11	14	– 11
12	– 15	16
13	25	8
14	9	7
15	– 9	10

(i) What is the beta for stock A?

(ii) What is the characteristic line for stock A? 5+5

(b) Discuss the relationship embodied in the security market line. Suggest an appropriate measure for the market risk premium and justify it. Show how the capital market line is a special case of the security market line. 3+2+1+4

(c) (i) Assume that a group of securities has the following characteristics: (a) the standard deviation of each security is equal to σ_A and (b) covariance of returns σ_{AB} is equal for each pair of securities in the group. What is the portfolio variance for a portfolio containing four securities which are equally weighted?

(ii) A portfolio consists of three securities, 1, 2 and 3. The proportions of these securities are $w_1 = 0{\cdot}3, w_2 = 0{\cdot}5$ and $w_3 = 0{\cdot}2$. The standard deviations of returns on these securities (in percentage terms) are $\sigma_1 = 6$, $\sigma_2 = 9$ and $\sigma_3 = 10$. The correlation coefficients among security returns are $\sigma_{12} = 0{\cdot}4$, $\sigma_{13} = 0{\cdot}6$, $\sigma_{23} = 0{\cdot}7$. What is the standard deviation of portfolio return? 5+5

(d) (i) A 10 percent coupon bond has a maturity of 12 years. It pays interest semi-annually. Its yield to maturity is 4 percent per half-year period. What is its duration?

(ii) A bond of Rs. 1,00,000 per five-year maturity with a 9 percent coupon rate (paid annually) currently sells at a yield to maturity of 8 percent. A portfolio manager wants to forecast the total return on the bond over the coming two years, as his horizon is two years. He believes that two years from now, three-year maturity bonds will sell at a yield of 7 percent and the coupon income can be reinvested in short-term securities over the next two years at a rate of 6 percent. What is the expected annualized rate of return over the two-year period?

5+5

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B.A/B.Sc. 5th Semester (Honours) Examination, 2022 (CBCS)

Subject : Mathematics

Course : BMH5DSE23

(Boolean Algebra and Automata Theory)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Define modular lattices with an example.

(b) Define regular language and give an example.

(c) Prove that every finite language is regular.

(d) State duality principle.

(e) Differentiate PDA and non-deterministic PDA.

(f) Give the formal definition of TM. What are the different types of TMs?

(g) Prove that there is a countably infinite number of context-free languages.

(h) Define co-atom.

(i) Define finite automata.

(j) In a finite Boolean algebra, prove that every non-zero element can be uniquely expressed as the sum of all atoms.

(k) What are the advantages of CFL?

(l) What are the differences between a finite automata and a Turing machine?

(m) What are the applications of context-free languages?

(n) List out applications of pumping lemma.

(o) Let L be a distributive lattice and $c, x, y \in L$. If in $L, c \wedge x = c \wedge y$ and $c \vee x = c \vee y$, then show that $x = y$.

2. Answer *any four* questions: 5×4=20

(a) If f is a function of three Boolean variables x, y, z defined by $f(x, y, z) = xy + y'$, express f in Disjunctive Normal form. 5

(b) Write a short note on Halting problem.
Prove that in a bounded distributive lattice an element can have at most one complement. 2+3

(c) What is pushdown automata?
A lattice is modular iff it satisfies $x \vee (y \wedge (x \vee z)) = x \vee (z \wedge (x \vee y))$. 2+3

(d) What is recursively enumerable language?

If L and M are regular languages, then prove that $L - M$ is also regular languages. 2+3

(e) Let B be a Boolean algebra and $x, y \in B$. If $x \wedge y = 0$ and $x \vee y = 1$, then prove that $x = y'$. 5

(f) Describe how can a Turing machine be made as a unary to binary converter. 5

3. Answer *any two* questions: 10×2=20

(a) (i) Construct the finite automata equivalent to the regular expression

$01[((10)^* + 11)^* + 0]^*1.$

(ii) What is regular expression? What are the applications of regular expressions and finite automata? 5+(1+2+2)

(b) (i) Define context-free language. State and prove Pumping lemma.

(ii) Using Pumping lemma, show that language

$L = \{a^n b^n c^n : n \geq 1\}$ is not a context-free language. (2+4)+4

(c) (i) What is context-free grammar?

Write context-free grammar for the language $L(G) = \{a^n b^n : n > 0\}$.

(ii) Prove that any context-free language is generated by a context-free grammar in Chomsky normal form. 5+5

(d) (i) Prove in a Boolean algebra:

(I) $a + a = a$

(II) $a \cdot a = a$

(ii) Prove that every Boolean algebra is atomic.

(iii) Show that a lattice is distributive iff following identity holds:

$(x \wedge y) \vee (y \wedge z) \vee (z \wedge x) = (x \vee y) \wedge (y \vee z) \wedge (z \vee x)$ (2+2)+3+3

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