B.A/B.Sc 6th Semester (Honours) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH6DSE41
(Bio Mathematics)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions from the following: 6×5=30

(a) What is a Malthus model? What is its drawback? 3+2
(b) Explaine Michaelis-Menten Kinetics. What is a half saturation conatant? 3+2
(c) Define a simple Prey predator system with logistic growth of prey. 5
(d) What do you understand by equilibrium point? Find the eqiulibrium point/s of the following model

$$\frac{dX}{dt} = rX(k - X) - aXY$$

$$\frac{dY}{dt} = -bY + cXY$$

where r, k, a, b and c are all positive constants. 2+3

(e) Define the Lotka-Volterra model for prey predator system and find corresponding steady state. 4+1
(f) What is intra species competition? Give an example of two species competition model. 3+2
(g) Define a two dimensional system of nonlinear difference equation. How do you obtain the fixed point/s from it? 3+2
(h) What is diffusion in mathematical model? Give an example of two species model with diffusion. 3+2

2. Answer any three questions from the following: 3×10=30

(a) Discuss the phase plane analysis of a two dimensional system when

(i) the eigenvalues are real and
(ii) the eigenvalues are complex conjugate. 5 + 5

(b) Define a SIR model with generalized assumptions and hence analysize the stability of its eqiulibrium points.

$4+6$

(c) Consider the following system

$$\frac{dx}{dt} = x(1-\frac{x}{k}) - axy$$

$$\frac{dy}{dt} = exy - py$$

where k ; a ; e and p are all positive constants.

(i) Find corresponding steady states and Jacobean matrix around any fixed point.

(ii) Discuss the stability of interior steady state only. (3+2)+5

(d) (i) Define equilibrium solution of a nonlinear partial differential equation. Determine the equilibrium solutions of

$$u_t = Du_{xx},\ t \in (0,\infty), x \in (0,L)$$

$$u(0,x) = u_0(x),\ x \in [0,L]$$

$$u(t,0) = 0 = u(t,L), t \in (0,\infty).$$

where D is the diffusion constant.

(ii) What do you mean by a traveling wave solution? Find the traveling wave solutions of the wave equation

$$u_{tt} = c^2 u_{xx}$$

where c is a constant. 5+5

(e) (i) Reduce a two species diffusion model into a linearized system around any specially uniform steady state.

(ii) Obtain the conditions for diffusive instability. 5+5

B.A/B.Sc 6th Semester (Honours) Examination, 2020 (CBCS)

Subject: Mathematics

Course: BMH6DSE42

(Differential Geometry)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions from the following: 6×5=30

(a) Define a regular curve. Prove that each reparametrization of a regular curve is regular. 1+4

(b) If γ is a space curve, then prove that its curvature is given by

$$\kappa = \frac{\|\ddot{\gamma} \times \dot{\gamma}\|}{\|\dot{\gamma}\|^3}$$

where '$\times$' indicates vector product and $\dot{\gamma} = \frac{d}{dt}(\gamma)$. 5

(c) Compute the curvature of the circular helix $\gamma(\theta) = (\mathrm{a}\cos\theta,\ a\sin\theta,\ b\theta)$, where $-\infty < \theta < \infty$ and a, b are constants. 5

(d) Prove that a curve on a surface is a geodesic if and only if its geodesic curvature is zero everywhere. 5

(e) If γ is a unit speed space curve with constant curvature and zero torsion, then prove that it is a circle. 5

(f) Determine the principal curvatures of the circular cylinder $\sigma(u,v) = (\cos v, \sin v, u)$ 5

(g) If $\gamma(t) = \sigma\big(u(t), v(t)\big)$ is a unit speed curve on a surface patch σ, prove that its normal curvature is given by

$$\kappa_n = L\,\dot{u}^2 + 2M\dot{u}\dot{v} + N\dot{v}^2,$$

where $Ldu^2 + 2M\,du\,dv + Ndv^2$ is the second fundamental form of σ and 'over-dot' denotes $\frac{d}{dt}$. 5

(h) Define a smooth surface. Prove that a plane is a smooth surface. 1+4

2. Answer any three questions from the following: 3×10=30

(a) Define torsion of a space curve. Prove that a space curve is a plane curve if an only if its torsion is zero everywhere on the curve. 2+8

(b) State and prove the fundamental theorem of a plane curve. 2+8

(c) Deduce Serret-Frenet Formulae for a space curve. 10

(d) Deduce the Gaussian curvature of a unit sphere S^2. 10

(e) State and prove Euler's theorem. 2+8

B.A/B.Sc 6th Semester (Honours) Examination, 2020 (CBCS)

Subject: Mathematics

Course: BMH6DSE43

(Mechanics-II)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions from the following: 6 x 5= 30

(a) Show that the Newton's second law of motion is invariant under the Galilean transformation. 5

(b) Obtain the fundamental equation in the form: grad $p = \rho \vec{F}$ for a fluid in equilibrium under a given system of external forces $\vec{F}$ per unit mass of the fluid, p and ρ denoting the fluid pressure and density respectively. 5

(c) What are meant by constraints on a dynamical system? Is the constraint relation $yz\frac{dx}{dt} + zx\frac{dy}{dt} + xy\frac{dz}{dt} = 0$ holonomic? Justify your answer. 2+3

(d) Obtain the relation between pressure and volume in an adiabatic change of a gas. 5

(e) Discuss the work-energy theorem of a system of many particles. 5

(f) Find the depth of the centre of pressure of a triangular area in terms of the depth of its vertices when the lamina is immersed in a homogeneous liquid keeping its plane vertical. 5

(g) Obtain the Lagrangian for the motion of a particle of unit mass moving in a central force field under inverse square law of force and also obtain an equation connecting the radial co-ordinate and time . 5

(h) A gas at uniform temperature is acted on by the following force

$$X = \frac{-x}{x^2+y^2+z^2},\ Y = \frac{-y}{x^2+y^2+z^2},\ Z = \frac{-z}{x^2+y^2+z^2}.$$

Find its density at the point (x, y, z). 5

2. Answer any three from the following questions. 3 x 10 = 30

(a) Define generalised co-ordinates of a dynamical system. Deduce Lagrange's equation of motion for a dynamical system of *n* degrees of freedom specified by *n* generalized co-ordinates $q_1, q_2, ..., q_n$ in a conservative field of force. 2+8

(b) State and prove any two properties of a stress quadric. 5+5

(c) (i) Show that the pressure at a point in a fluid in equilibrium is the same in every directions.

(ii) Assuming the atmosphere to be in convective equilibrium, find the expression for absolute temperature T at a height z in the form $\frac{T}{T_0} = 1 + \frac{r-1}{r}\frac{z}{H}$ when the variation of gravity is neglected. 5+5

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(d) Obtain the expression for kinetic energy of a system of N particles using generalized velocity. 10

(e) (i) Prove that the surfaces of equi-pressure are intersected orthogonally by the lines of force.

(ii) Show that the surface of separation of two liquids of different densities which do not mix, at rest under gravity, is a horizontal plane. 5+5