B.Sc. 3rd Semester (Honours) Examination, 2018 (CBCS)

Subject : Physics

Paper : CC-V

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* of the following questions: 2×5=10

(a) Suppose we expand $f(x) = \sin^3 x, 0 \le x \le 2\pi$, in a Fourier sine series. How many non-zero coefficients will be there and what will be their values?

(b) Find the differential equation of a family of circles passing through the origin and having centres on the positive x-axis.

(c) What do you mean by ordinary point, regular singular point and irregular singular point of an ordinary linear homogeneous second order differential equation?

(d) Starting from the generating function of Legendre polynomials show that $P_l(-1) = (-1)^l$.

(e) Write down Laguerre's differential equation and also write the generating function of Laguerre's polynomials.

(f) If *erf* (x) be the error function of x, show that $\int_a^b e^{-x^2}dx = \frac{\sqrt{\pi}}{2}[erf(b) - erf(a)]$.

(g) If the probability of a defective bolt is 0·1, find the mean and the standard deviation for the distribution bolts in a total of 400.

(h) Form the differential equation by eliminating the arbitrary function f and g from $z = f(x+at) + g(x-at)$

2. Answer *any two* of the following questions: 5×2=10

(a) Find the Fourier series of the function

$$\begin{aligned} f(x) &= -1 \; for \; -\pi < x < -\frac{\pi}{2} \\ &= 0 \; for \; -\frac{\pi}{2} < x < \frac{\pi}{2} \\ &= +1 \; for \; \frac{\pi}{2} < x < \pi \end{aligned}$$

5

(b) Prove that

(i) $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x}J_n(x)$

(ii) $J_{n-1}(x) - J_{n+1}(x) = 2J_n'(x)$

Where the symbols have their usual meaning. 5

(c) The density function of a random variable x is $f(x) = Ke^{-2x^2+10x}$

Find the upper 5% point of the distribution of the means of a random sample of size 25 from the above population. 5

(d) Solve the differential equation: $\frac{d^2x}{dt^2} + \frac{g}{l}x = \frac{g}{l}L$

where g, l and L are constants subject to the conditions at $t = 0$, $x = a$ and $\frac{dx}{dt} = 0$. 5

3. Answer *any two* of the following questions: 10×2=20

(a) (i) Find the Fourier series expansion of the function

$$\begin{aligned} f(x) &= -\pi \; for \; -\pi < x < 0 \\ &= x \; for \; 0 < x < \pi \end{aligned}$$

and deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

(ii) If $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}\right)$,

Prove that

$$\int_{-l}^{+l}[f(x)]^2\,dx = l\left[\frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)\right].$$

(4+1)+5=10

(b) (i) Derive Rodrigue's formula for the Legendre polynomials

$$P_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l$$

Hence find the value of $P_2(x)$.

(ii) Prove that

$$\int_0^{\pi}\left|P_l(\cos\theta)\right|^2 \sin\theta\, d\theta = \frac{2}{2l+1}$$

where the symbols have their usual meaning. (5+1)+4=10

(c) (i) Establish Dirichlet's integral $\iint_D x^{l-1}y^{m-1}dx\,dy = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m+1)}h^{l+m}$

Where D is the domain $x \geq 0, y \geq 0$ and $x + y \leq h$.

(ii) Show that the area in the first quadrant enclosed by the curve

$\left(\frac{x}{a}\right)^{\alpha} + \left(\frac{y}{b}\right)^{\beta} = 1, \alpha > 0, \beta > 0$, is $\frac{ab}{\alpha+\beta}\frac{\Gamma\left(\frac{1}{\alpha}\right)\Gamma\left(\frac{1}{\beta}\right)}{\Gamma\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)}$ 5+5=10

(d) Show that Laplace's equation in two dimensions can be written in the polar form as

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

Solve the above equation by the method of separation of variables with the following boundary conditions:

(i) $u(r, 0) = 0; 0 \leq r \leq a$

(ii) $u(r, \pi) = 0; 0 \leq r \leq a$

and (iii) $u(a, \theta) = T; 0 \leq \theta \leq \pi$ 2+8=10

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