B.Sc. 1st Semester (Honours) Examination, 2017 (CBCS)

Subject : Physics

Paper : CC-I

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group–A

1. Answer *any five* questions: 2×5=10

(a) Show that the magnitude of a vector remains invariant under rotations of the coordinate system.

(b) The independent random variables X and Y have the probability density functions $g(x) = e^{-x}$ and $h(y) = 2e^{-2y}$ respectively. Calculate the probability that X lies in the interval $1 < X \leq 2$ and Y lies in the interval $0 < Y \leq 1$.

(c) Define the Dirac delta function $\delta(x)$. Find the value of $\int_{-\infty}^{+\infty} \delta(x)\, e^{ikx}\, dx$.

(d) Use binomial expansion to evaluate $1/\sqrt{4.2}$ to four places of decimals.

(e) Find from first principles the first derivative of $(2x+3)^2$.

(f) Given $\Phi = xy + yz + zx$, find $\vec{\nabla}\Phi$ at (1, 2, 3) and the directional derivative of Φ at the same point in the direction towards the point (3, 4, 4).

(g) Define Wronskian of two differentiable functions and hence find the Wronskian for $a(x) = x^2$ and $b(x) = Sin\ 6x$.

(h) State Stokes theorem of vector calculus.

Group–B

Answer *any two* questions: 5×2=10

2. (a) What do you mean by variance?

(b) A point P is chosen at random on the circle $x^2 + y^2 = 1$. The random variable X denotes the distance of P from (1, 0). Find the mean and variance of X and the probability that X is greater than its mean. 1+(1+1+2)=5

3. If $\vec{F} = 3xy\hat{\imath} - y^2\hat{\jmath}$, evaluate $\int_c \vec{F}.\vec{dr}$, where c is the curve in the xy plane $y = 2x^2$ from (0, 0) to (1, 2). 5

4. (a) If $u\vec{F} = \vec{\nabla}v$, where u and v are scalar fields and $\vec{F}$ is a vector field, show that $\vec{F}.(\vec{\nabla}\times\vec{F}) = 0$.

(b) Evaluate $\vec{\nabla}.(\Phi\vec{A})$, where $\vec{A} = 2x\hat{\jmath} + 3y\hat{\jmath} - 4z\hat{k}$ and $\Phi = xyz$. 2+3=5

5. The temperature of a point (x, y) on a unit circle is given by $T(x,y) = 1 + xy$. Find the temperature of the two hottest points on the circle. 5

Group–C

Answer *any two* questions: 10×2=20

6. (a) State and explain divergence theorem.

(b) Verify the above theorem for a vector field $\vec{A} = xy^2\hat{\imath} + y^3\hat{\jmath} + y^2z\hat{k}$ and the surface of the cuboid defined by $0 < x < 1, 0 < y < 1$ and $0 < z < 1$.

(c) Find also the Curl of the above vector $(\vec{A})$. 2+6+2=10

7. (a) Express the vector field $\vec{A} = (x\hat{\imath} + y\hat{\jmath} + 4\hat{k})/\sqrt{x^2 + y^2 + z^2}$ in cylindrical and spherical coordinates.

(b) The vertices of a triangle are located at (4, 1, –3), (–2, 5, 4) and (0, 1, 6). Find the three angles of the triangle by the use of vectors only.

(c) Identify the surfaces (i) $\vec{r}.\vec{u} = l$ and (ii) $\vec{r}.\vec{u} = m|\vec{r}|$ for $-1 \leq m \leq +1$. where l and m are fixed scalars and $\vec{u}$ is a fixed unit vector. (2+2)+(1+1+1)+(1+2)=10

8. (a) Solve: $(\cos^2 x + y \sin 2x)\frac{dy}{dx} + y^2 = 0$

(b) Determine whether the $(y + z)dx + x\,dy + x\,dz$ is an exact differential or not.

(c) Solve: $x(1 - 2x^2y)\frac{dy}{dx} + y = 3x^2y^2$ given that $y(1) = \frac{1}{2}$ 4+2+4=10

9. (a) What is meant by "homogeneous" and "linear" second order differential equation?

(b) Find the general solution of the equation: $\frac{d^2y}{dx^2} + 4y = 2\cos 2x$

(c) Find out the extremum points of the function $f(x) = 2x^3 - 3x^2 - 12x + 4$. Also plot the function to show those points. 2+5+3=10