

B.Sc. 5th Semester (Honours) Examination, 2022 (CBCS)

Subject : Physics

Course : DSE-1(1)

(Advanced Mathematical Physics)

Time: 2 Hours

Full Marks: 40

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

All symbols have their usual meanings.

SECTION-I

1. Answer any five questions:

2×5=10

- The vectors \vec{r} in real three dimensional space V_3 is transformed to \vec{r}' by an operator A as follows: $\vec{r}' = A\vec{r} = \vec{a} \times \vec{r}$, where \vec{a} is a constant vector. Show that A is linear.
- Define Hermitian matrix. Check whether the matrix $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ is Hermitian or not?
- What is the relation between the trace of a matrix and the eigenvalues of that matrix? What is the relation between determinant of a matrix and eigenvalues of that matrix?
- Write down Lorentz transformation equations in the context of Einstein's Special Theory of Relativity in matrix form.
- Define inner product of two tensors each of order one.
- If $A = \lambda x^i$ for all values of the independent variables $x^1, x^2, x^3, \dots, x^n$. and λ_i 's are constant, show that $\frac{\partial}{\partial x^j} (\lambda_i x^i) = \lambda_j$.
- Write down expression of the Christoffel 3 index symbols of the first and the second kind.
- Write down Quotient Law of tensors.

SECTION-II

(Answer any two questions.)

5×2=10

2. Linear transformation T on R^3 acts on standard basis and gives us ordered triplets of real numbers, defined by

$$T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}; T \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \text{ and } T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

then compute $T \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$

3. Exponential of a matrix can be written as power series of the given matrix (provided the series converges):

Using this hint prove that for a matrix $M = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, one can obtain $e^M = \begin{pmatrix} e^a & 0 \\ 0 & e^b \end{pmatrix}$

4. Show that in Rotational Mechanics moment of inertia (\mathbf{I}) is actually a tensor of order 2. Here $\vec{L} = \vec{r} \times \vec{p} = \mathbf{I}\vec{\omega}$
5. (a) Show that all diagonal elements of an anti-symmetric matrix are zero.
 (b) Show that in an n -dimensional space S_n , an anti-symmetric (skew symmetric) covariant tensor of second order (i.e. anti-symmetric matrix) has at most $\frac{n(n-1)}{2}$ different components.

SECTION-III

(Answer any two questions.)

10×2=20

6. Solve the given coupled differential equations using matrix method:

$$\frac{dx}{dt} = 2x - 3y$$

$$\frac{dy}{dt} = y - 2x$$

Subject to initial conditions $x(0) = 8, y(0) = 3$.

10

7. Write down the Caley-Hamilton's theorem and using this theorem find the inverse of the matrix

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

10

8. Given $(ds)^2 = (dr)^2 + r^2(d\theta)^2 + r^2 \sin^2 \theta (d\phi)^2$ defines the line element in a spherical polar coordinates. Find the values of $[13, 3]$ and $\left\{ \begin{matrix} 3 \\ 13 \end{matrix} \right\}$

5+5=10

9. The line element is given

$$ds^2 = g_{pq} dx^p dx^q = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1 dx^2 + 4dx^2 dx^3$$

Find the

- (a) Conjugate tensor g^{ij} (also called second fundamental tensor) and
 (b) g (determinant of g_{ij}) corresponding to the metric.

5+5=10

B.Sc. 5th Semester (Honours) Examination, 2022 (CBCS)**Subject : Physics****Course : DSE-1(2)****(Medical Physics)****Time: 2 Hours****Full Marks: 40***The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words
as far as practicable.**Symbols and abbreviations have their usual meanings.***Section-I****1. Answer any five questions:****2×5=10**

- Give example of one 1D and one 2D synovial joints in human body.
- Find out the degrees of freedom of human thumb joints.
- What is 'tachycardia'?
- What is Residual Volume (RV) of human lungs? How it is related with TLC?
- The heart rate of a person is 120 pulses/min. Calculate the action time and resting time of heart muscle.
- What do you mean by 1 seivert?
- What are K-alpha and K-beta emission lines in case of characteristic X-ray?
- What is NMR imaging?

Section-II**2. Answer any two questions:****5×2=10**

- A person has a systolic pressure 150 mmHg, diastolic pressure 100 mmHg, heart rate 90/min. Calculate the work done by the heart and the efficiency of the heart (lower left half) if the energy consumed is 6 watt. **5**
- The mass of the pulmonary blood of a person is 1.5 kg. Find the approximate mass of that person and the mass of his systematic blood. **5**
- Write down short notes (*any two*):
 - Conformal Radiation Therapy (CRT)
 - Brachytherapy
 - Radioactive tracers **2.5+2.5=5**
- What do you mean by visual field loss in case of glaucoma? A person can clearly see the objects lying between 25cm and 2m from his eyes. Find out the nature and power of lenses that can be used for his corrected vision. **1+4=5**

Section-III

3. Answer any two questions:

10×2=20

- (a) Find the expression of power exerted during a throw using elbow motion (in case of human body). If the torque generated is 36.45 N-m in the case with a 3 inch diameter muscle, then calculate the average power generated during a throw. 6+4=10
- (b) What is ultrasound imaging? The velocity of ultrasound in a commercial transducer probe is 4000 m/s. If a vibration frequency of 5 MHz is desired, what would be the crystal thickness? Define resting potential and action potential. Draw the electrical circuit analogues to small axon, hence find out the energy required to recharge 1 meter length of non-myelinated axon. Where, $C = 3 \times 10^{-7} \text{F/m}$. 1+3+2+4=10
- (c) What is bremsstrahlung? Calculate the wavelength of X-ray that undergoes second order reflection at 15° from the face of a cubic crystal of KCl. Where crystal density = 1.98 gm/cm^3 and Avogadro No. = 6.02×10^{23} . In a self-rectified X-ray machine circuit, line voltage is 220V. If voltage is stepped by a transformer of turn ratio 500 : 1, then what will be the resultant peak voltage (applied) of X-ray tube? What is the disadvantage of self-rectified X-ray machine? 1+4+3+2=10
- (d) What is respiratory cycles? Describe PSV and SIMV in case of mechanical ventilator. Explain the basic principle of computed Tomography Scan (multiple sclice). How it is different from X-ray imaging? 2+4+3+1=10